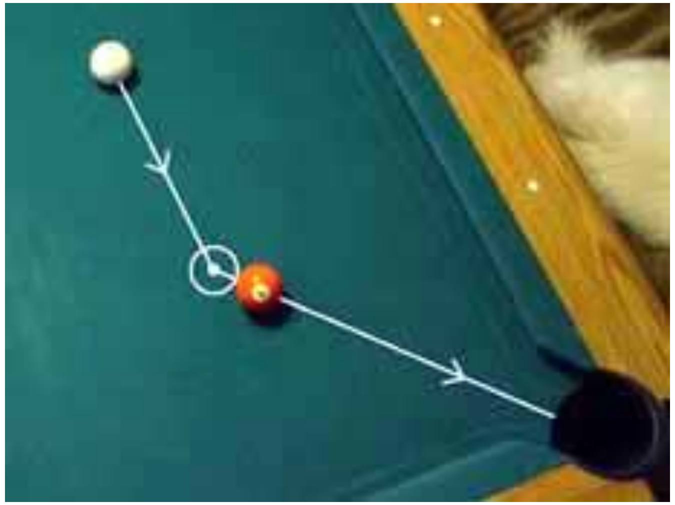
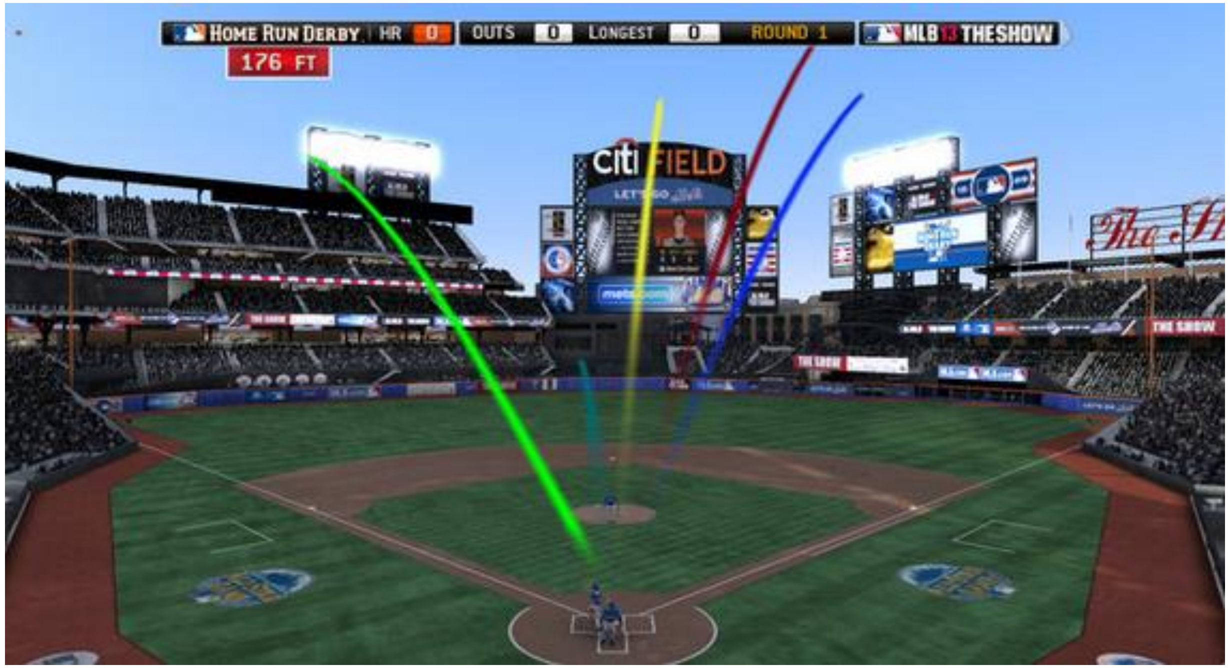
Vectors in the Plane

Section 6.3

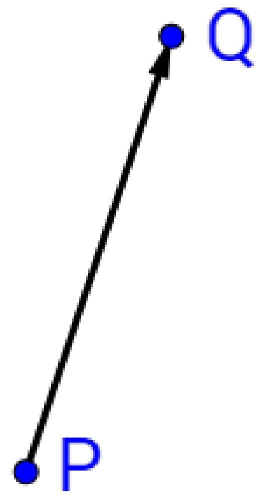
Real World Vectors





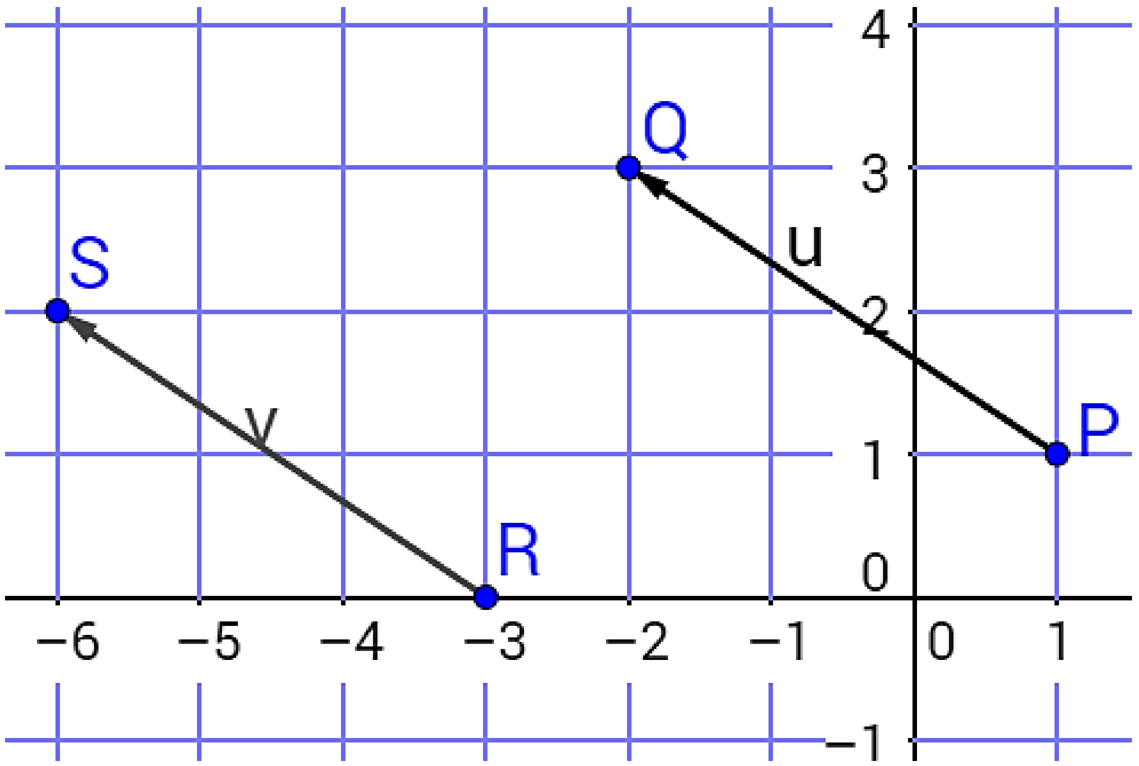
Directed Line Segments

When describing force and velocity, it's important to represent their magnitude and direction. A directed line segment $\overrightarrow{PQ}$ has an initial point P and a terminal point Q. Its magnitude, or length can be found using the distance formula. The set of all directed line segments in a plane equivalent to $\overrightarrow{PQ}$ is a vector **v**.



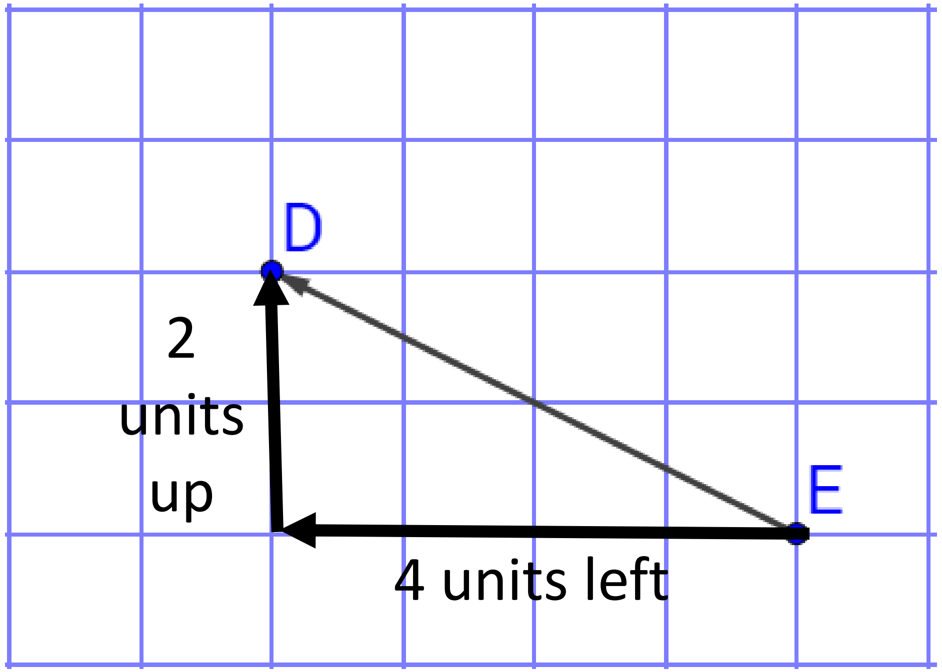
Show Two Vectors are Equivalent

Let **u** be the directed line segment from P(1, 1) to Q(-2, 3), and let **v** be the directed line segment from R(-3, 0) to S(-6, 2). Show that **u** and **v** are equivalent.

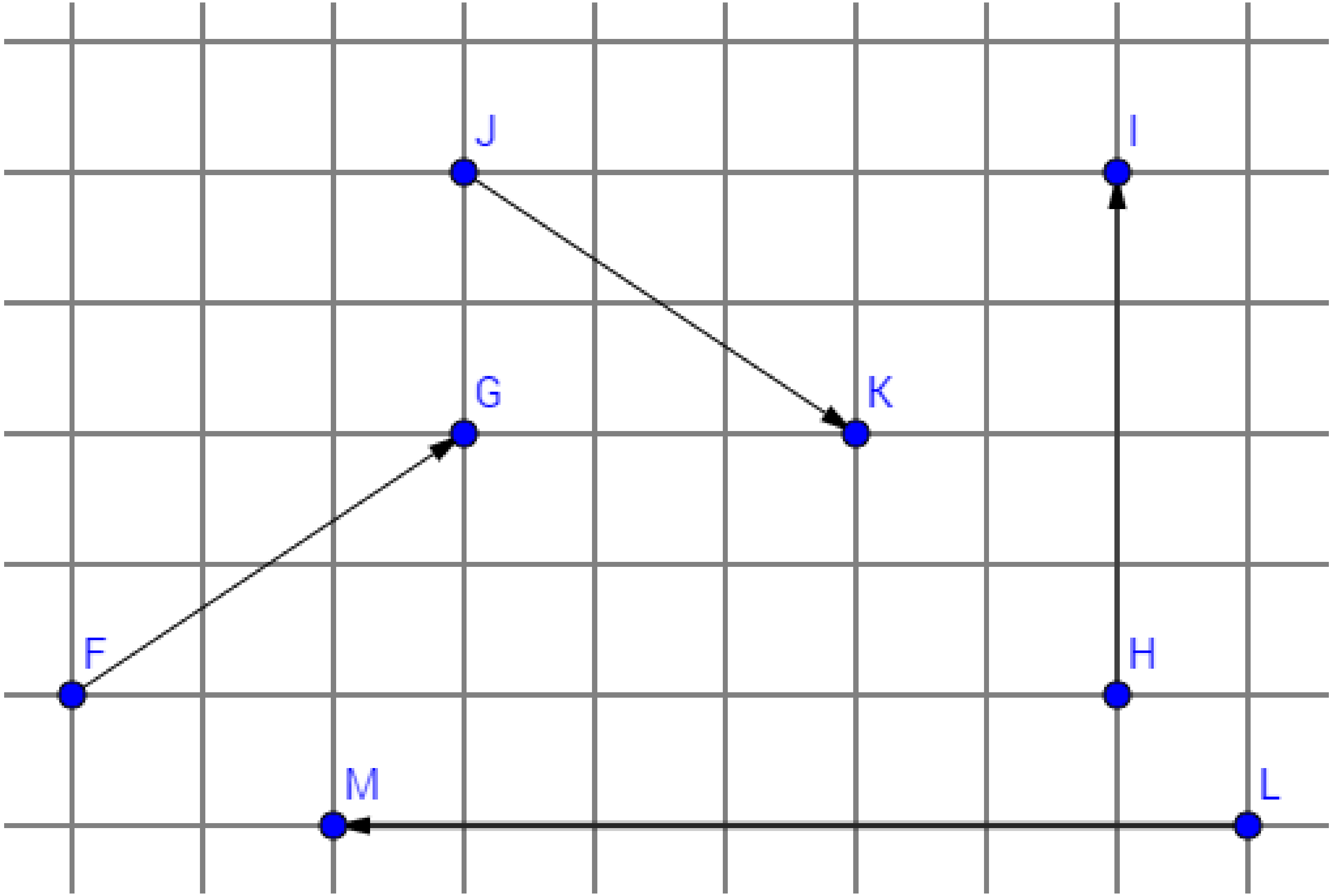


Vectors

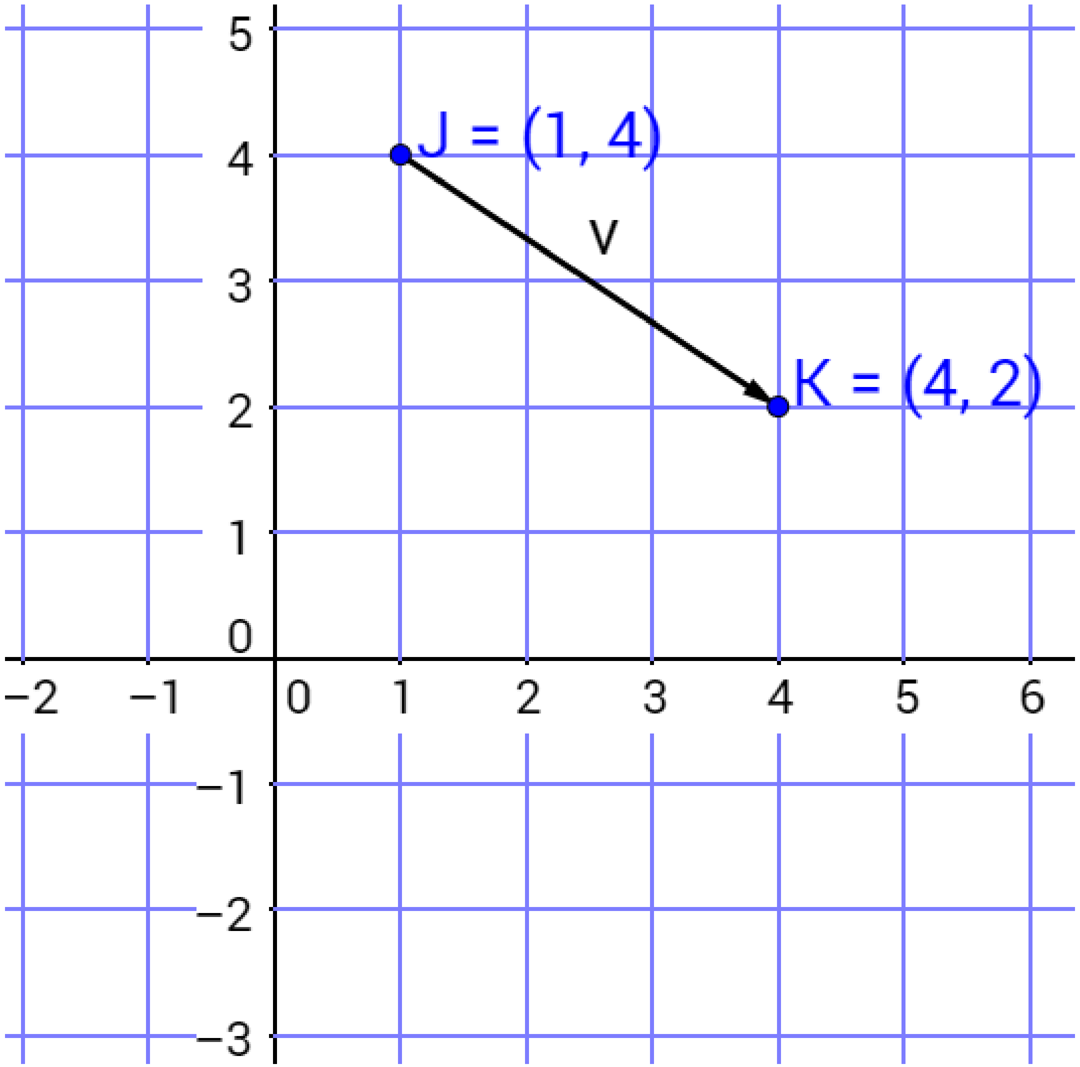
A vector whose initial point is the origin (0, 0) is said to be in standard position. The terminal point of a vector in standard position is at (v_1, v_2) and is called the component form of the vector. The component form of $\overrightarrow{ED}$ is $\langle -4, 2 \rangle$.



Name the vectors. Write their component forms.



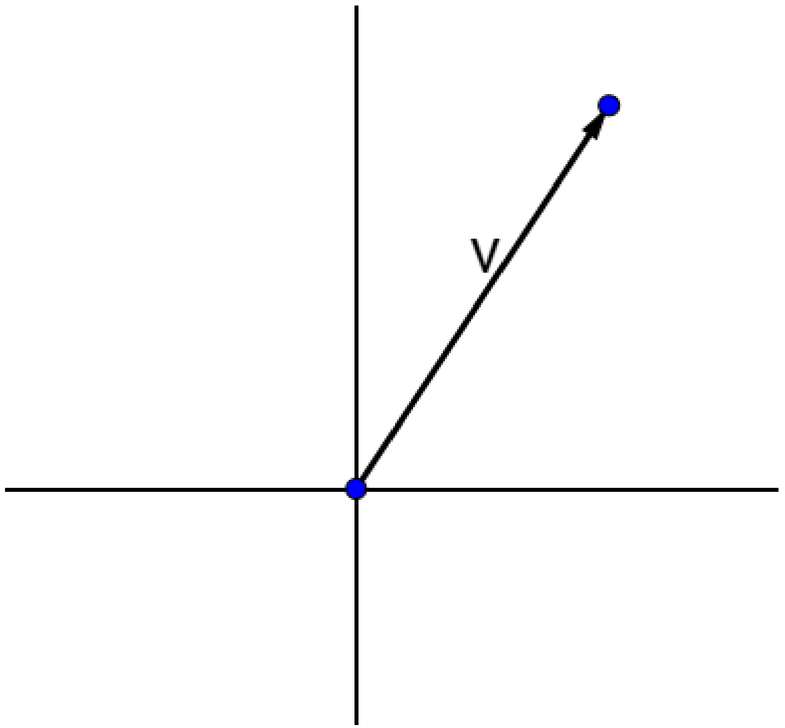
Find the component form of the vector **v** with the initial point J(1, 4) and terminal point K(4, 2).



Magnitude of a Vector

The component form of a vector **v** in standard form is v = $\langle a, b \rangle$. The magnitude or length of **v** is

$$\|v\| = \sqrt{a^2 + b^2}$$



Find the magnitude of u = $\langle -3, 8 \rangle$

Adding Vectors

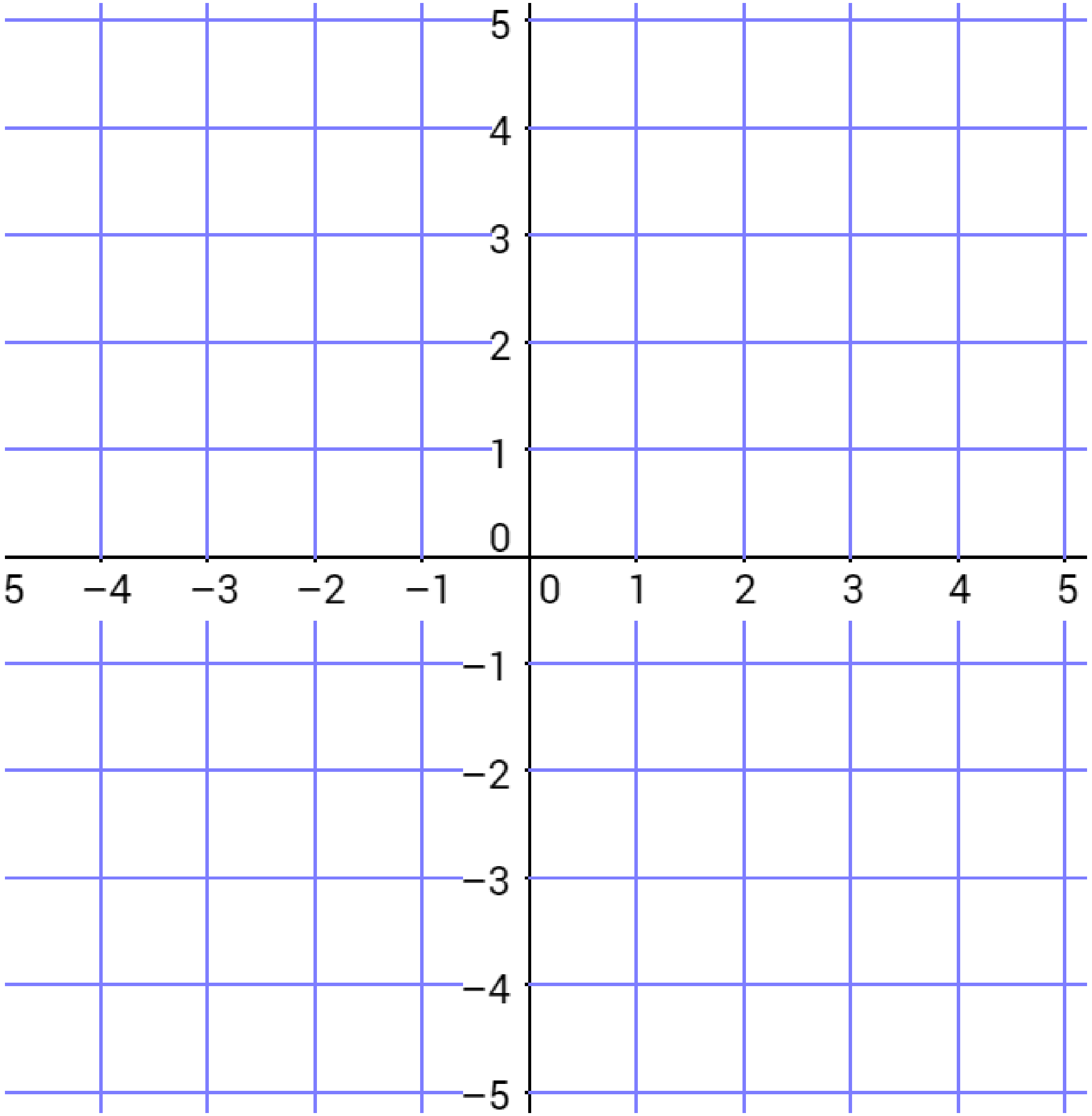
The sum or resultant of two vectors is also a vector. Vectors **u** and **v** can be added using tail-to-head addition or parallelogram addition.

Given **u** = $\langle -2, 1 \rangle$ and **v** = $\langle 4, 3 \rangle$

Find **u** + **v**.

Find **u** – **v**.

Find 4**u**.



Refer to p. 421

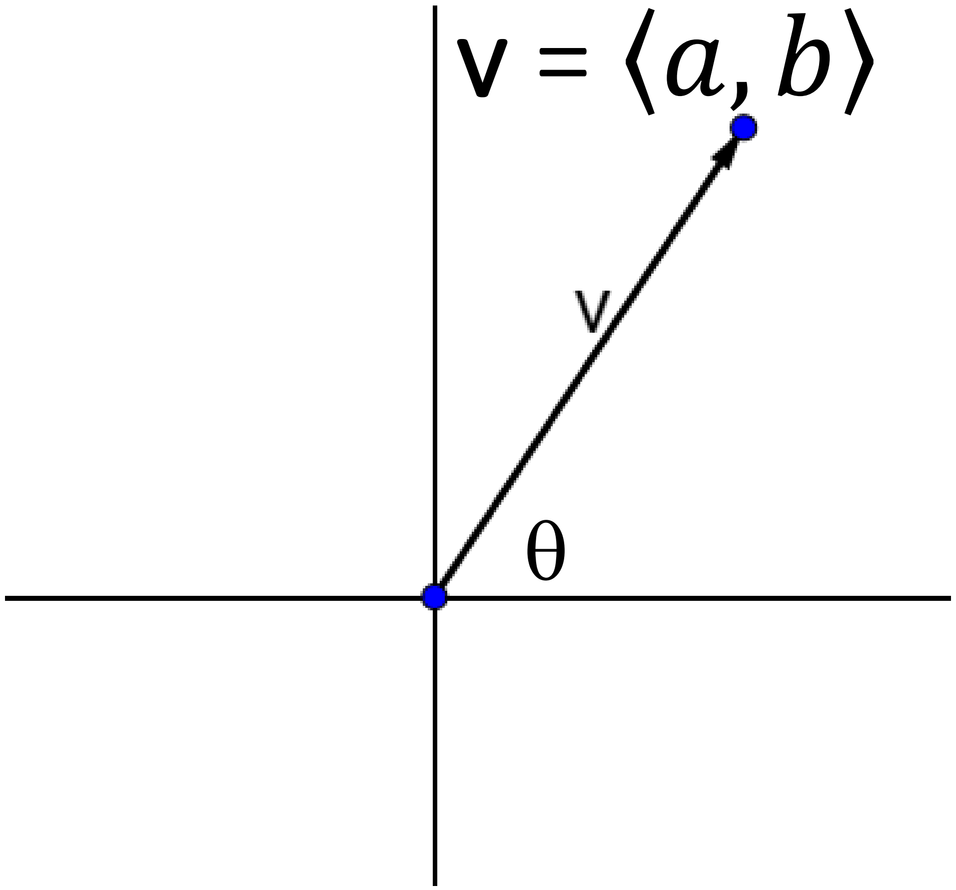
Properties of Vector Addition and Scalar Multiplication

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors and let c and d be scalars. Then the following properties are true.

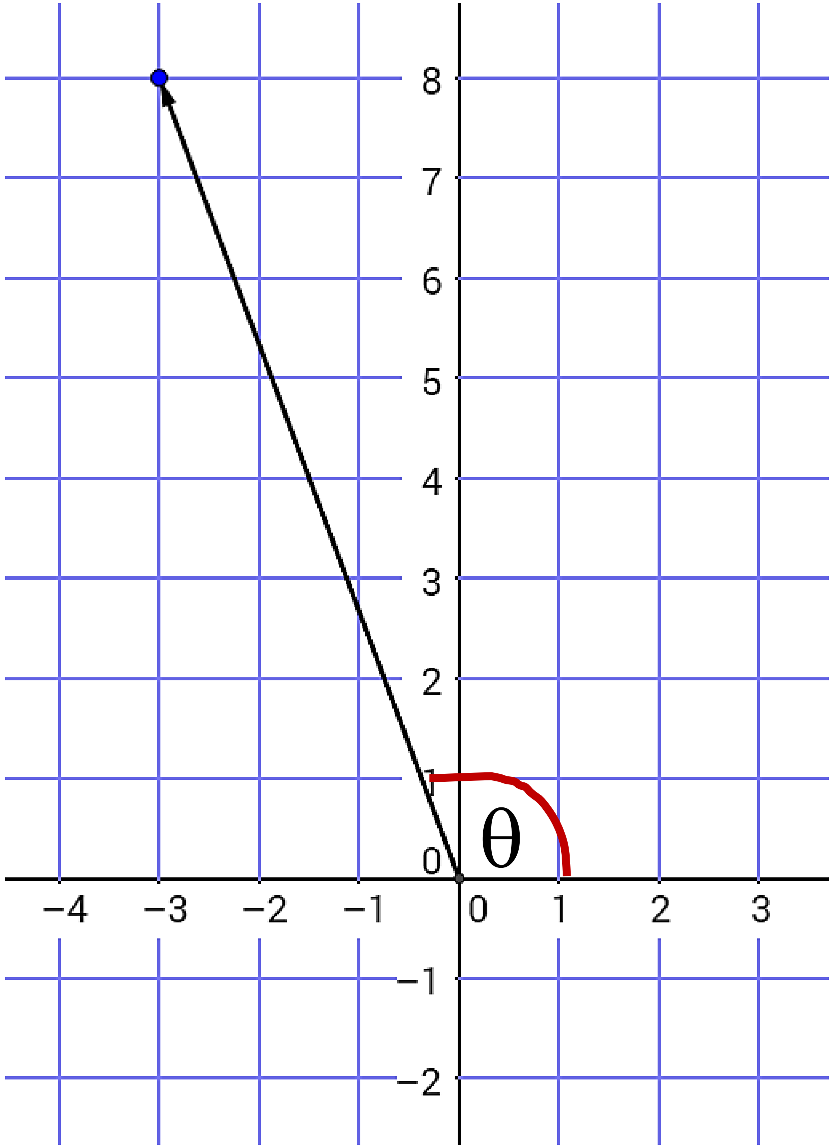
1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(d\mathbf{u}) = (cd)\mathbf{u}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $1(\mathbf{u}) = \mathbf{u}, \quad 0(\mathbf{u}) = \mathbf{0}$
9. $\|c\mathbf{v}\| = |c| \, \|\mathbf{v}\|$

The direction angle is the measure of the angle between the vector and the positive x-axis.

Find the direction angle for $\langle 5, 4 \rangle$.



Find the direction angle for $\langle -3, 8 \rangle$.



Unit Vector = vector with a magnitude of 1

u = unit vector = $\left(\frac{1}{\|v\|}\right) v$

Note that u is a scalar multiple of v. The vector u is a vector with a magnitude of 1 and is in the same direction as v.

Find the unit vector of v = $\langle -3, 8 \rangle$

Standard Unit Vectors and Linear Combination Notation

Standard unit vectors are written as **i** and **j**

i = $\langle 1, 0 \rangle$

horizontal component

j = $\langle 0, 1 \rangle$

vertical component

$\langle -3, 8 \rangle$ = -3i + 8j

4i – 3j = $\langle 4, -3 \rangle$

Section 6.3 p. 427; 11 – 42 x 3’s, 53, 56

Vectors in the Plane

Day 2

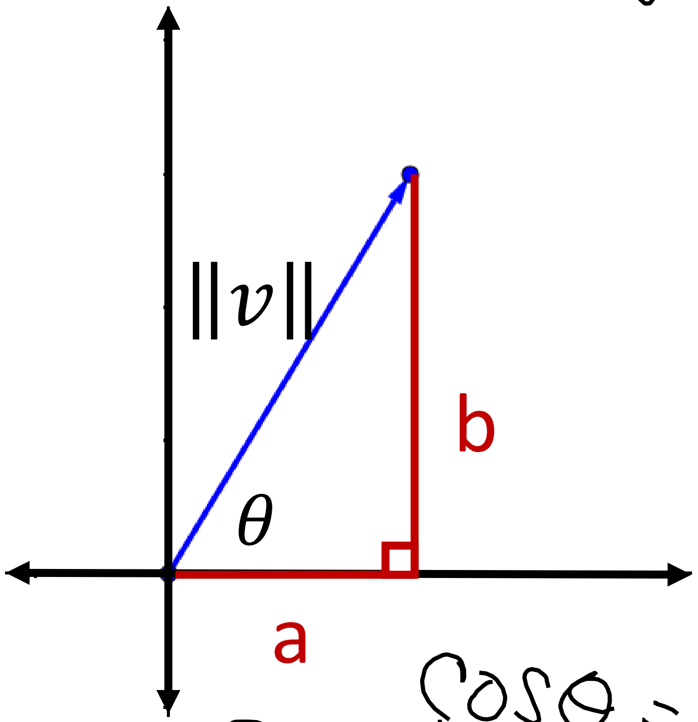
Writing Component/Linear Combination Form Given Magnitude and Direction Angle.

$\mathbf{v} = \langle a, b \rangle$

$ai + bj$

$\tan\theta = \frac{b}{a}$

$||v|| = \sqrt{a^2 + b^2}$



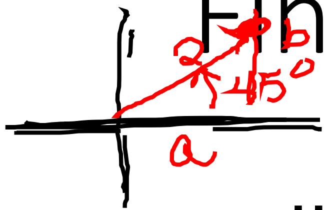
$\cos\theta = \frac{a}{||v||}$

$\sin\theta = \frac{b}{||v||}$

$$\mathbf{v} = \langle ||v|| cos\theta, ||v|| sin\theta \rangle = ||v||\ cos\theta \mathbf{i} + ||v|| sin\theta \mathbf{j}$$

$\langle a, b \rangle$

Find the component form of **v**.



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a. $\|v\| = 2, \theta = 45°$

$\langle \|v\|\cos\theta, \|v\|\sin\theta \rangle$

$\langle 2\cos 45, 2\sin 45 \rangle$

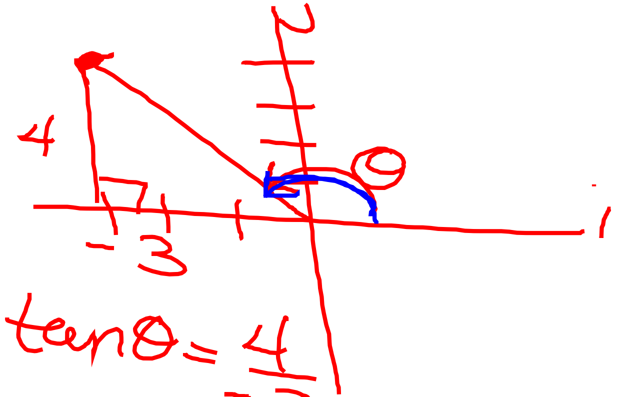
$\langle 2\frac{\sqrt{2}}{2}, 2\frac{\sqrt{2}}{2} \rangle$

$\langle \sqrt{2}, \sqrt{2} \rangle$

$\|u\| = 5$

b. $\|v\|$ = 3, **v** is in the direction of -3i + 4j

$\langle -3, 4 \rangle$

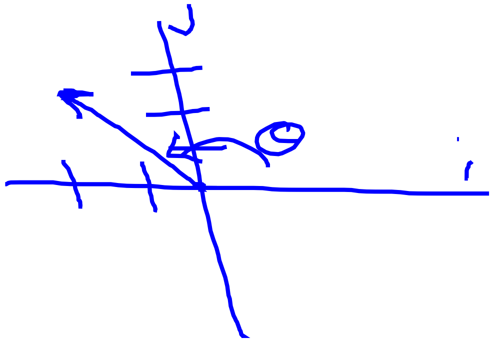


$\tan\theta = \frac{4}{-3}$

$-53.13°$

$126.87°$

$\theta_v = 126.87°$



$\|v\|\cos\theta\, i + \|v\|\sin\theta\, j$

$3(\cos 126.87)i + 3\sin(126.87)j$

$-1.8i + 2.4j$

$\langle a, b \rangle + \langle c, d \rangle = \langle a+b, c+d \rangle$

Find the component form of the sum of **u** and **v**.

- $||u||$ = 4; $\theta_{\mathrm{u}} = 60°$
- $||v||$ = 5; $\theta_{\mathrm{v}} = 90°$

$\langle 4\cos 60, 4\sin 60 \rangle$

$\langle 4(\frac{1}{2}), \frac{4\sqrt{3}}{2} \rangle$

$\langle 2, 2\sqrt{3} \rangle$

$\langle 5\cos 90, 5\sin 90 \rangle = \langle 0, 5 \rangle$

$\langle 2, 2\sqrt{3}+5 \rangle$

Applying Vectors

Forces with magnitudes of 2000 newtons and 900 newtons act on a machine part at angles of 30° and -45° respectively with the x-axis. Find the direction and magnitude of the resultant of these forces.

$\|u\| = 2000$ $\quad \theta_u = 30^\circ$

$\|v\| = 900N$ $\quad \theta_v = -45^\circ$

$\langle 2000\cos 30^\circ, 2000\sin 30^\circ \rangle$

$\langle 2000\frac{\sqrt{3}}{2}, 2000(\frac{1}{2}) \rangle$

$\langle 1000\sqrt{3}, 1000 \rangle$

$\langle 900\cos -45, 900\sin -45^\circ \rangle$

$\langle 900\frac{\sqrt{2}}{2}, 900(-\frac{\sqrt{2}}{2}) \rangle$

$\langle 450\sqrt{2}, -450\sqrt{2} \rangle$

$\langle 1000\sqrt{3} + 450\sqrt{2}, 1000 - 450\sqrt{2} \rangle$

(a) (b)

$\|u+v\| = 2396.19N$

$\theta = 8.72^\circ$

Velocity

A gun with a muzzle velocity of 1200 feet per second is fired at an angle of $6°$ with the horizontal. Find the vertical and horizontal components of the velocity.

Section 6.3 p. 428; 49, 52, 63-65, 67, 70, 73, 75, 76, 84, 85