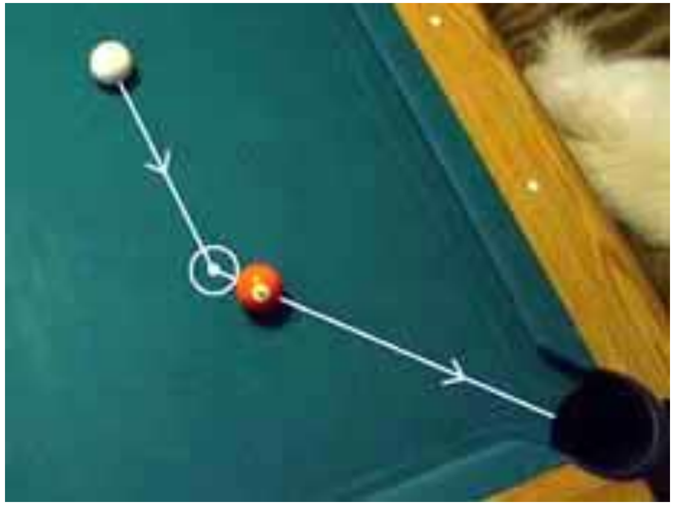
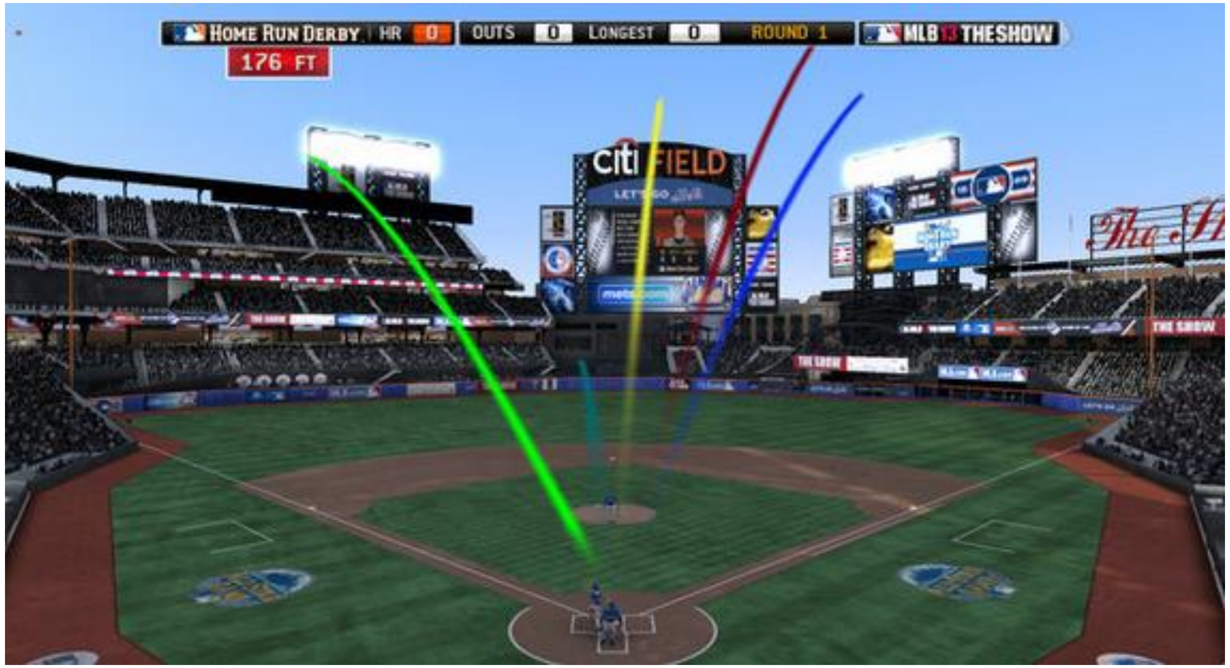
Vectors in the Plane

Section 6.3

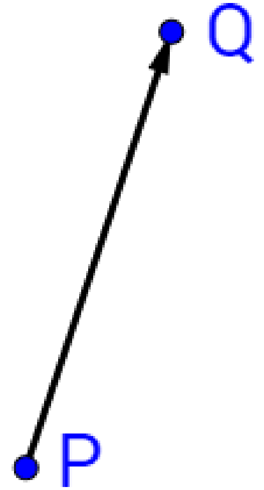
Real World Vectors





Directed Line Segments

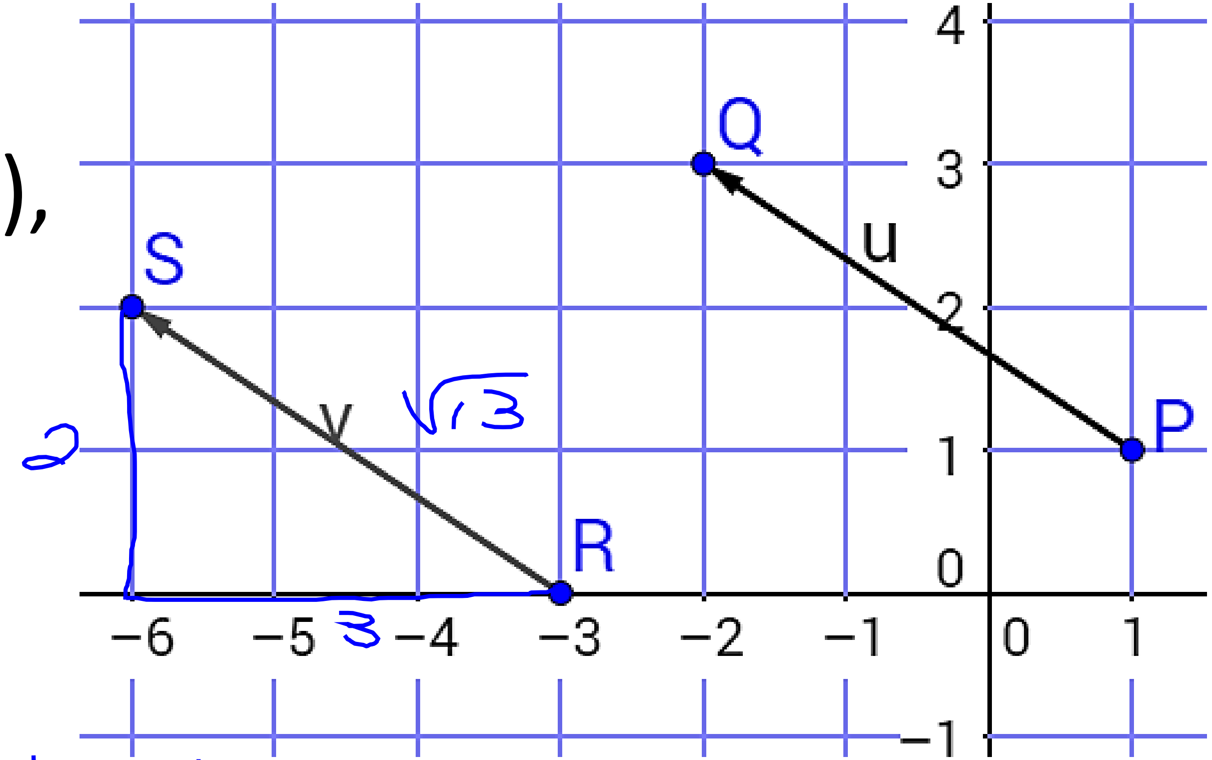
When describing force and velocity, it's important to represent their magnitude and direction. A directed line segment $\overrightarrow{PQ}$ has an initial point P and a terminal point Q. Its magnitude, or length can be found using the distance formula. The set of all directed line segments in a plane equivalent to $\overrightarrow{PQ}$ is a vector **v**.



Show Two Vectors are Equivalent

$\frac{y_2 - y_1}{x_2 - x_1} = m$

Let **u** be the directed line segment from P(1, 1) to Q(-2, 3), and let **v** be the directed line segment from R(-3, 0) to S(-6, 2). Show that **u** and **v** are equivalent.



$PQ = \sqrt{(-2-1)^2 + (3-1)^2}$

$\sqrt{9+4} = \sqrt{13}$

$RS = \sqrt{(-6+3)^2 + (2-0)^2} = \sqrt{13}$

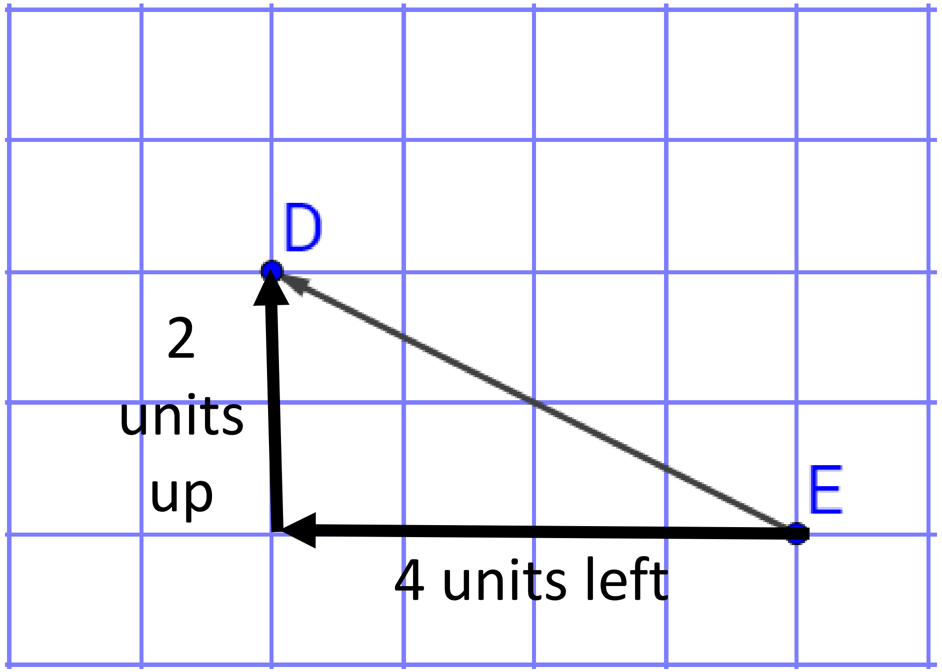
$\sqrt{9+4}$

$\overrightarrow{RS}$ has slope of $\frac{2}{-3}$

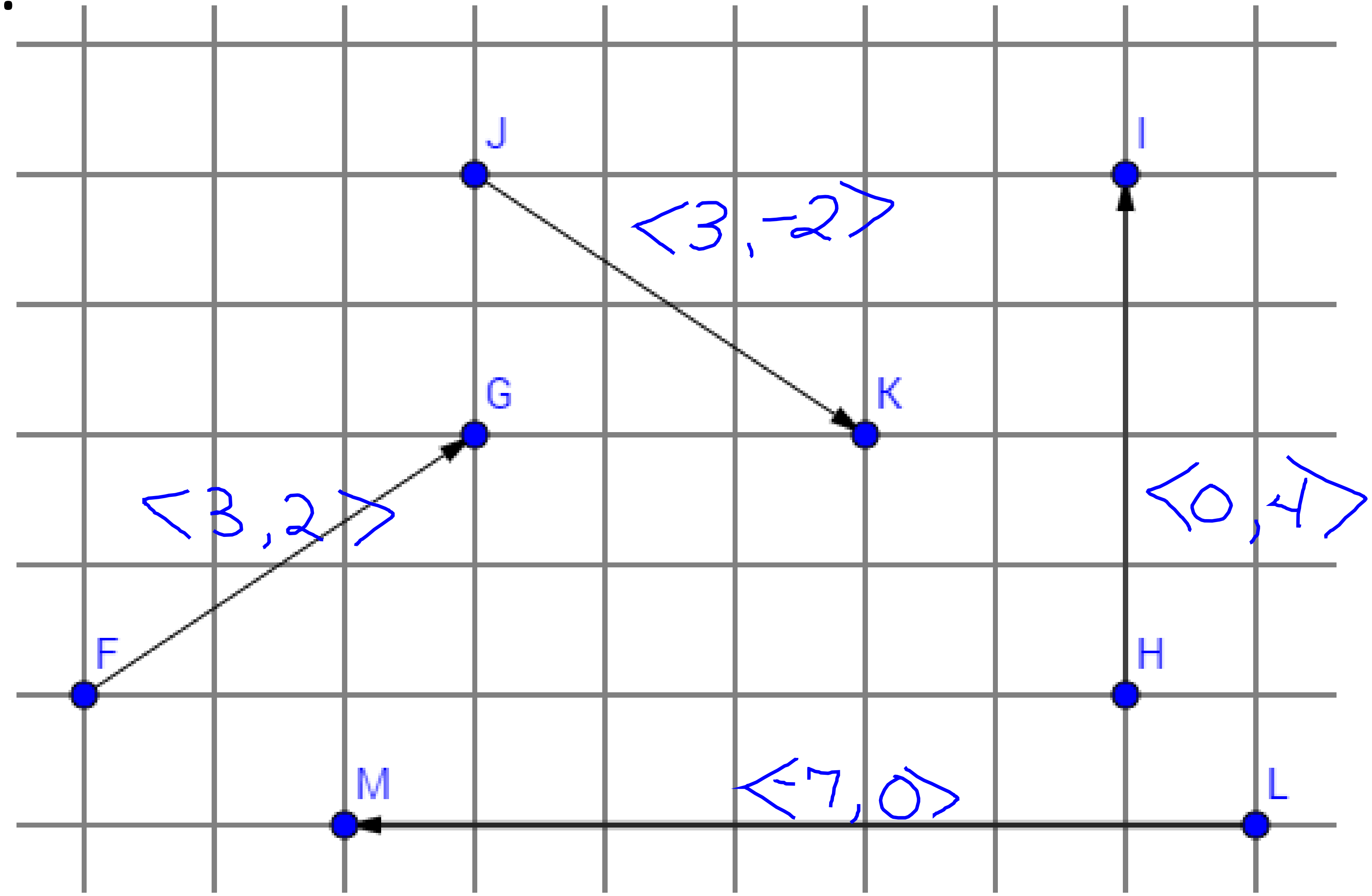
$\overrightarrow{PQ}$ has a slope of $2/-3$

Vectors

A vector whose initial point is the origin (0, 0) is said to be in standard position. The terminal point of a vector in standard position is at (v_1, v_2) and is called the component form of the vector. The component form of $\overrightarrow{ED}$ is $\boxed{\langle -4, 2 \rangle.}$



Name the vectors. Write their component forms.

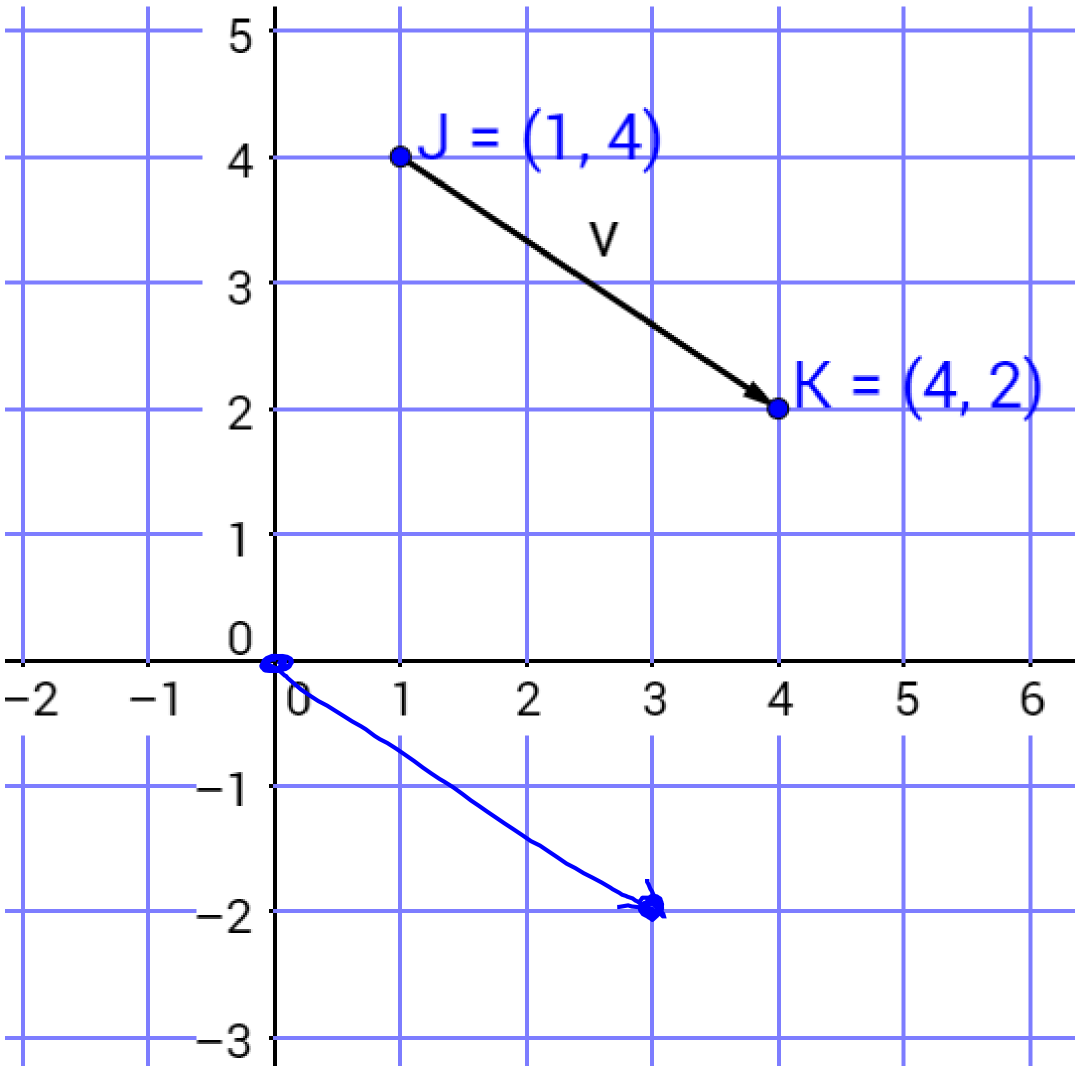


Find the component form of the vector **v** with the initial point J(1, 4) and terminal point K(4, 2).

terminal − initial

$\langle 4-1, 2-4 \rangle$

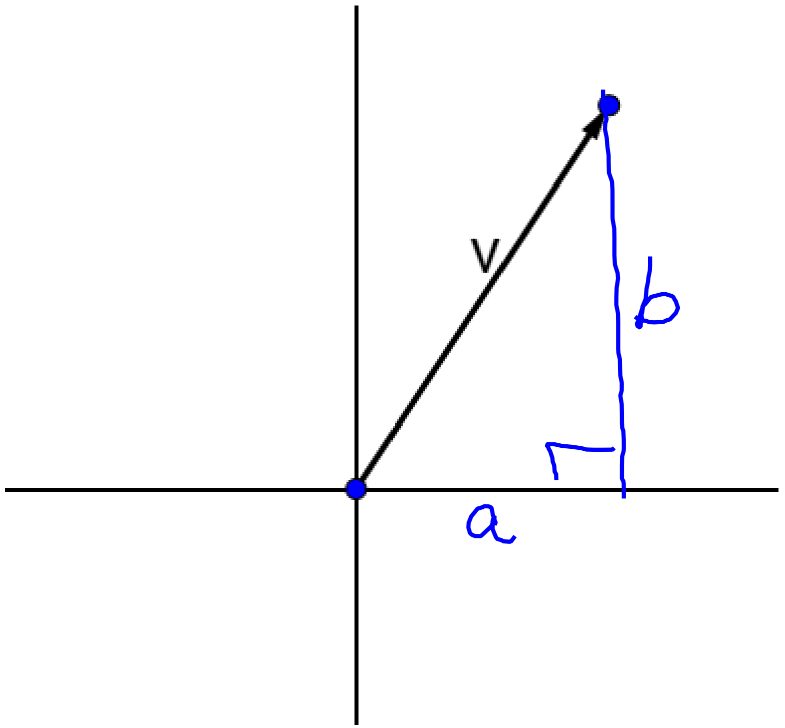
$\langle 3, -2 \rangle$



Magnitude of a Vector

The component form of a vector **v** in standard form is v = $\langle a, b \rangle$. The magnitude or length of **v** is

$$\|v\| = \sqrt{a^2 + b^2}$$



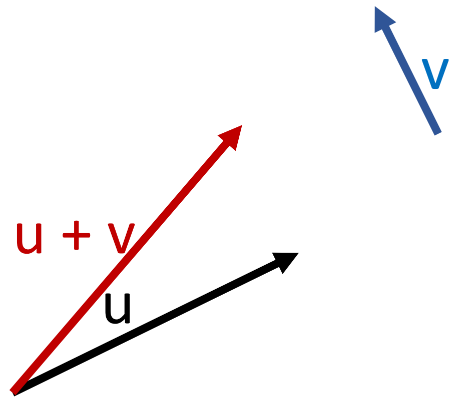
Find the magnitude of u = $\langle -3, 8 \rangle$

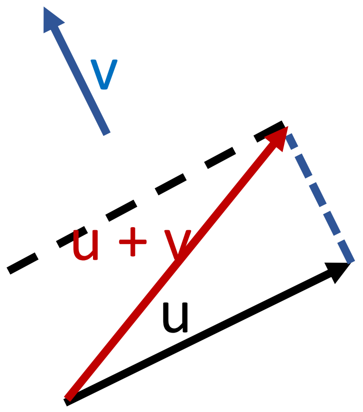
$$\|u\| = \sqrt{(-3)^2 + (8)^2}$$

$$\sqrt{9+64} = \sqrt{73}$$

Adding Vectors

The sum or resultant of two vectors is also a vector. Vectors **u** and **v** can be added using tail-to-head addition or parallelogram addition.





Given $\mathbf{u} = \langle -2, 1 \rangle$ and $\mathbf{v} = \langle 4, 3 \rangle$

Find $\mathbf{u} + \mathbf{v}$.

$\langle -2+4,\ 1+3 \rangle$

$\langle 2, 4 \rangle$

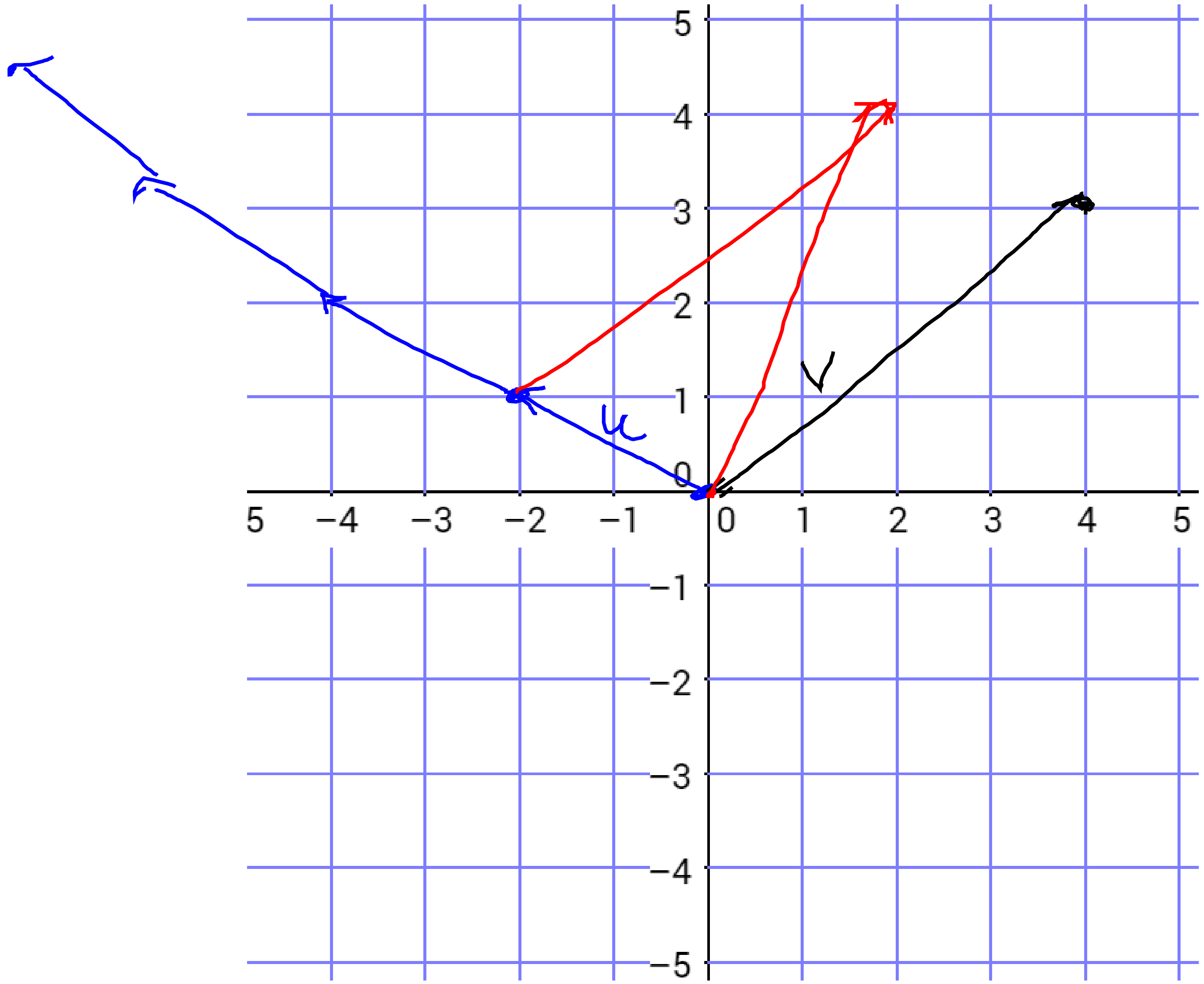
Find $\mathbf{u} - \mathbf{v}$.

$\langle -2+{}^{-}4,\ 1+{}^{-}3 \rangle$

$\langle -6, -2 \rangle$

Find $4\mathbf{u}$.

$\langle -8, 4 \rangle$



Refer to p. 421

Properties of Vector Addition and Scalar Multiplication

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors and let c and d be scalars. Then the following properties are true.

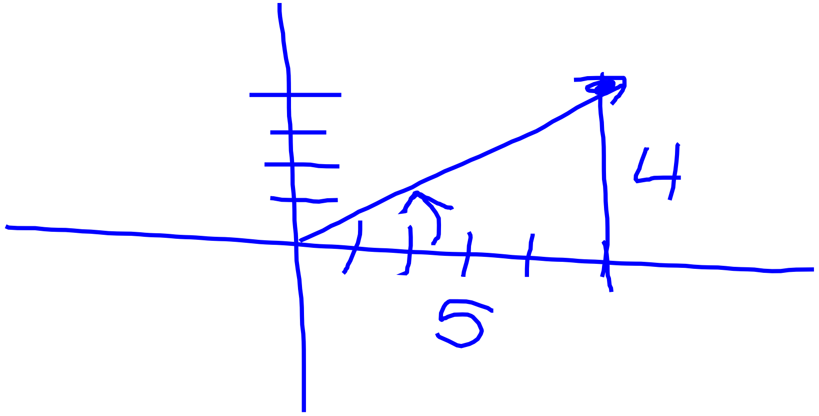
1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(d\mathbf{u}) = (cd)\mathbf{u}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $1(\mathbf{u}) = \mathbf{u}, \quad 0(\mathbf{u}) = \mathbf{0}$
9. $\|c\mathbf{v}\| = |c|\,\|\mathbf{v}\|$

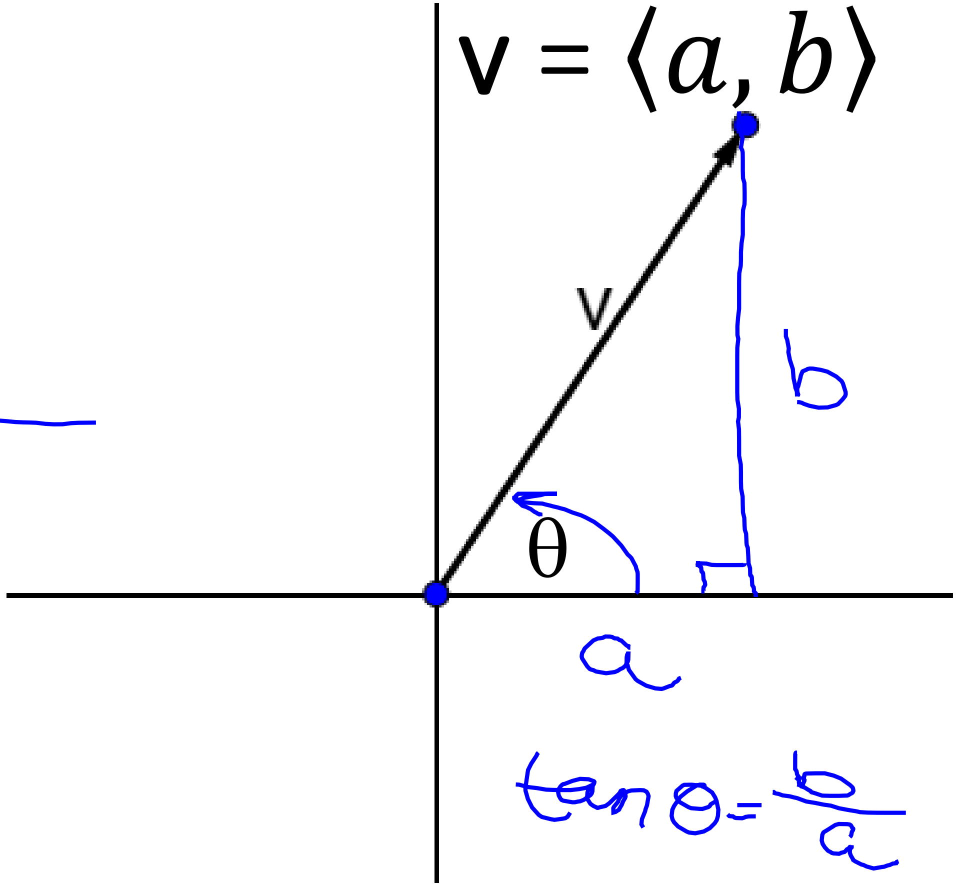
The direction angle is the measure of the angle between the vector and the positive x-axis.

Find the direction angle for $\langle 5, 4 \rangle$.

$\tan\theta = \frac{4}{5}$

$\theta = 38.66$



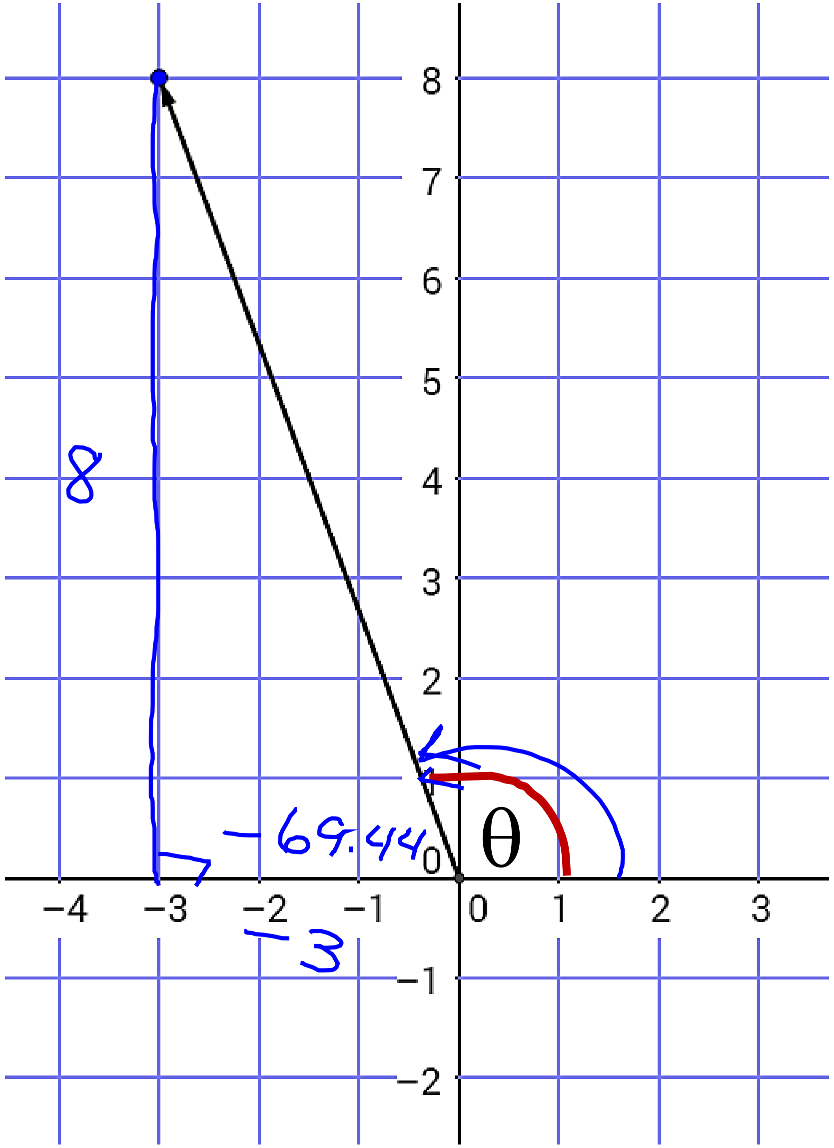


Find the direction angle for $\langle -3, 8 \rangle$.

$\tan \theta = \frac{8}{-3}$

$\theta = -69.44$

$\theta = 110.56^\circ$



Unit Vector = vector with a magnitude of 1

u = unit vector = $\left(\frac{1}{\|v\|}\right) v$

Note that u is a scalar multiple of v. The vector u is a vector with a magnitude of 1 and is in the same direction as v.

Find the unit vector of v = $\langle -3, 8 \rangle$

$\|v\| = \sqrt{73}$ $\quad \left\langle \frac{-3}{\sqrt{73}}, \frac{8}{\sqrt{73}} \right\rangle \quad \left\langle \frac{-3\sqrt{73}}{73}, \frac{8\sqrt{73}}{73} \right\rangle$

Standard Unit Vectors and Linear Combination Notation

Standard unit vectors are written as **i** and **j**

i = $\langle 1, 0 \rangle$ $\langle -3, 8 \rangle$ $-3i$

horizontal component

j = $\langle 0, 1 \rangle$ $\langle -3, 8 \rangle$ $+8j$

vertical component

$\langle -3, 8 \rangle$ = -3i + 8j

4i – 3j = $\langle 4, -3 \rangle$

Section 6.3 p. 427; 11 – 42 x 3’s, 53, 56