Trigonometric Functions: The Unit Circle

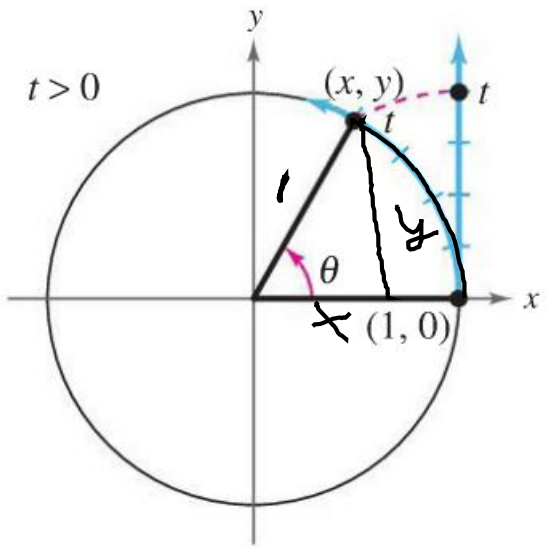
Section 4.2

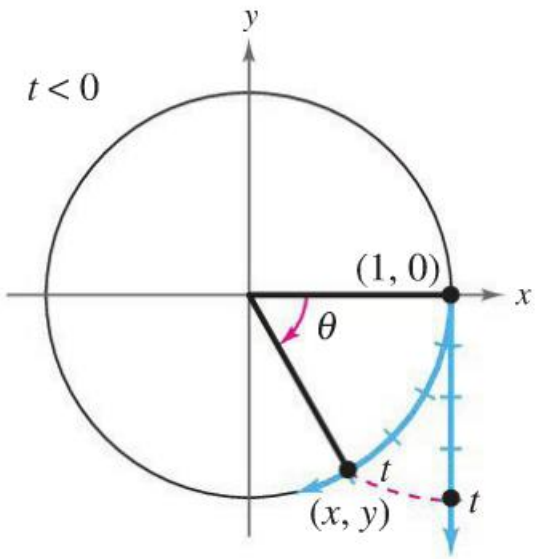
Definitions of Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x}, \quad x \neq 0$$

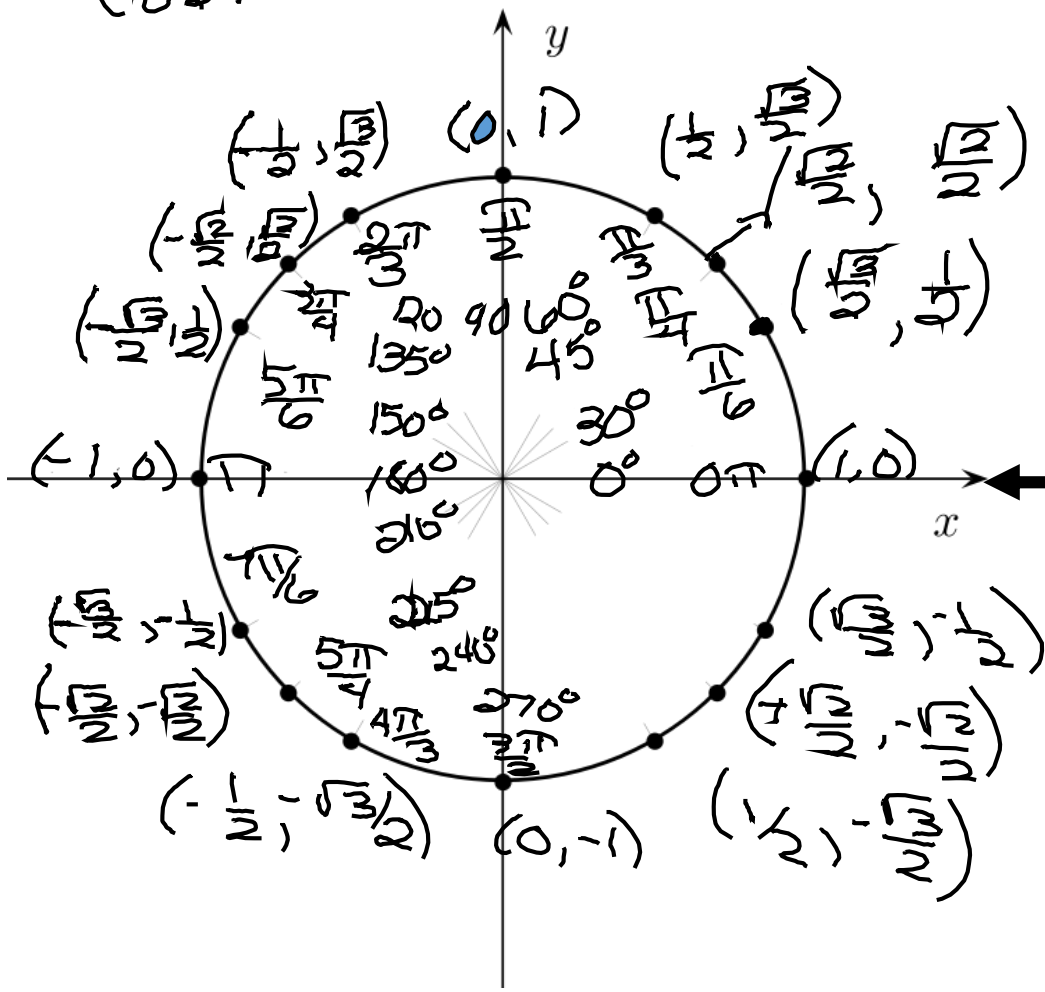
$$\csc t = \frac{1}{y}, \quad y \neq 0 \qquad \sec t = \frac{1}{x}, \quad x \neq 0 \qquad \cot t = \frac{x}{y}, \quad y \neq 0$$

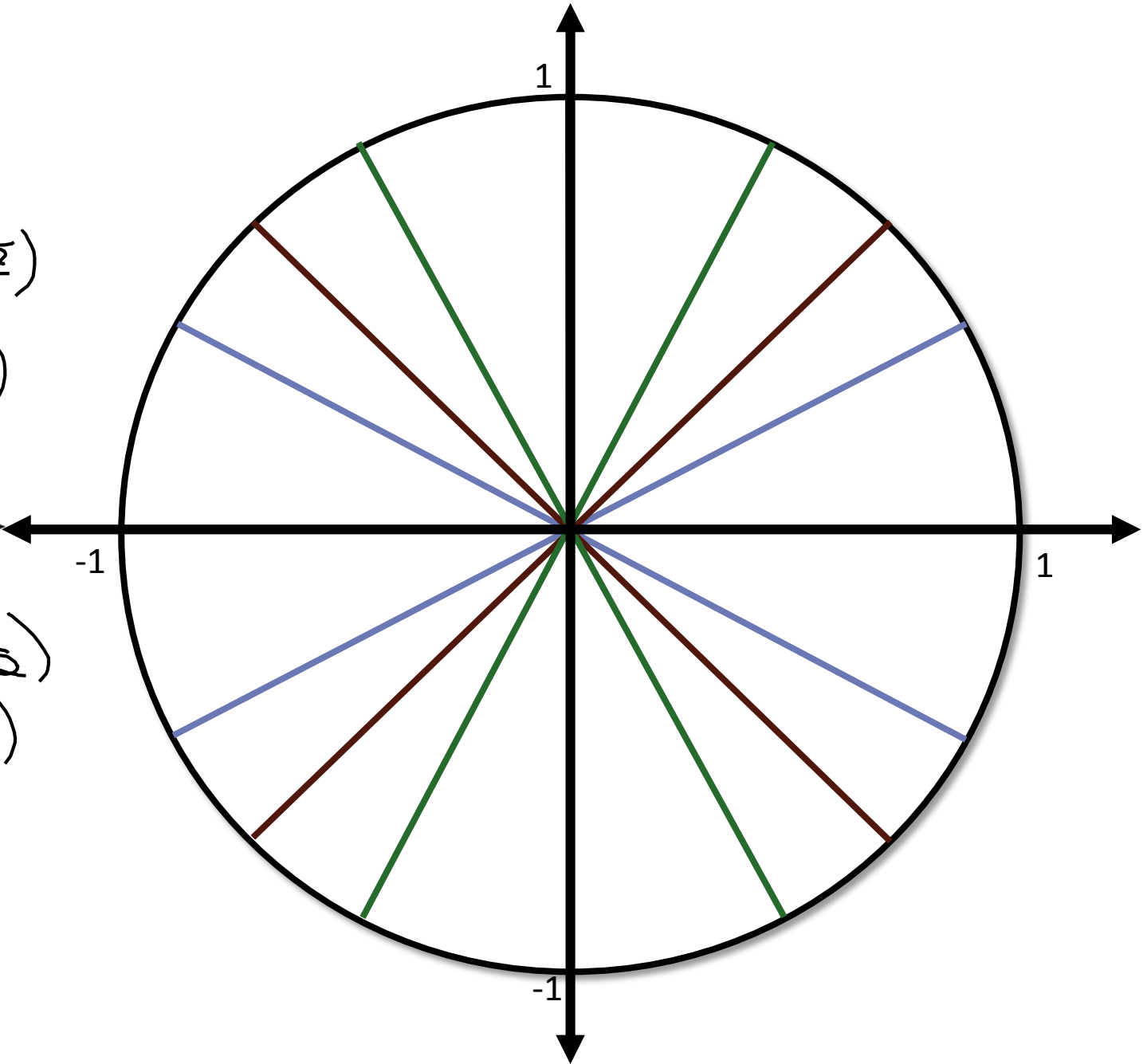




$\sin t = y$ $\cos t = x$

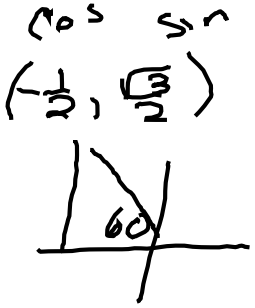
$\sin 30 = \frac{1}{2}$ $\cos 30 = \frac{\sqrt{3}}{2}$





Evaluate the six trig functions at each real number.

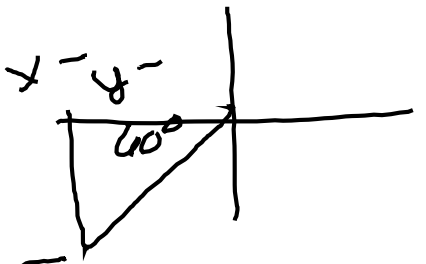
a. $t = \frac{2\pi}{3}$



$\sin t = \frac{\sqrt{3}}{2}$ $\cos t = -\frac{1}{2}$ $\tan t = -\sqrt{3}$

$\csc t = \frac{2\sqrt{3}}{3}$ $\sec\theta = -2$ $\cot\theta = -\frac{\sqrt{3}}{3}$

b. $t = \frac{4\pi}{3}$

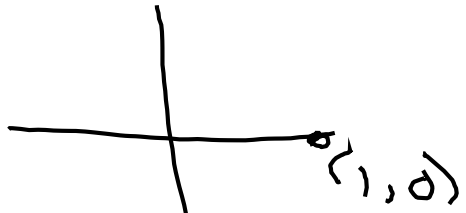


$\sin t = -\frac{\sqrt{3}}{2}$

$\cos t = -\frac{1}{2}$

$\tan t = \sqrt{3}$

c. $t = 2\pi$



$\sin t = 0$

$\cos t = 1$

$\tan\theta = 0$

$\csc\theta =$ und.

$\sec\theta = 1$

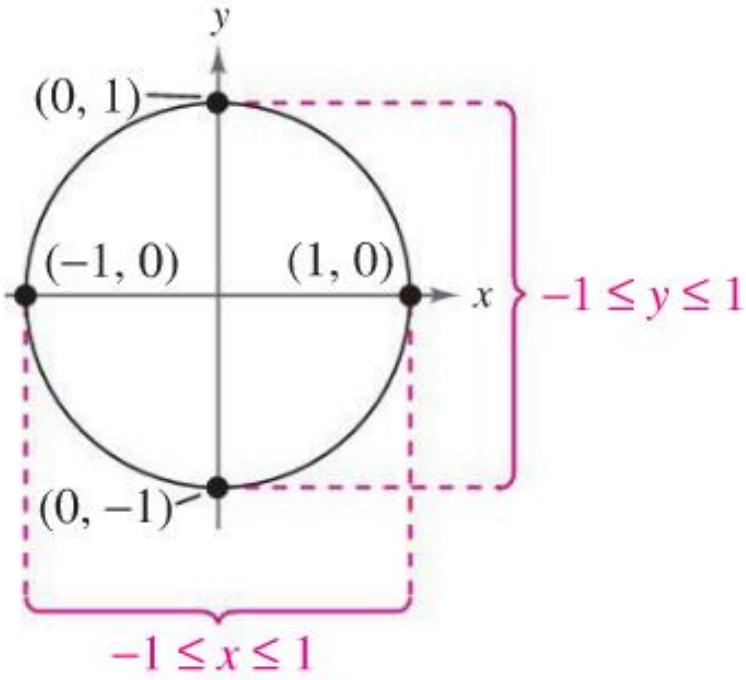
$\cot\theta =$ und.

d. $t = -\frac{2\pi}{3}$

Domain and Range of Sine and Cosine

Because (x, y) is on the unit circle, $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

$[-1, 1]$ $[-1, 1]$



Even and Odd Trig Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd*.

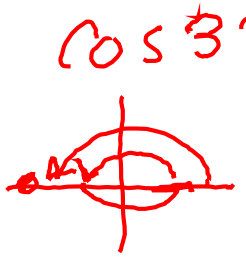
$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$

$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$

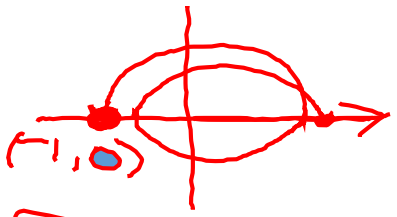
Find the following.

a. $\cos\frac{9\pi}{3}$

$\cos 3\pi$



$\cos 3\pi$



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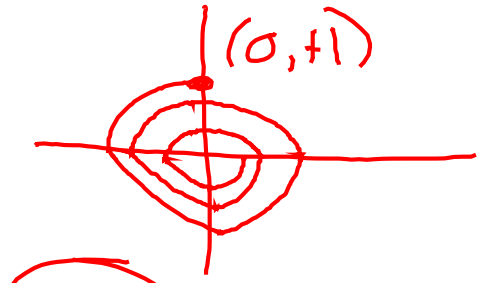
b. $\sin\left(-\frac{11\pi}{2}\right)$

$-\sin\frac{11\pi}{2}$



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c. If $\tan(t) = \frac{2}{3}$, find $\tan(-t)$.

$-\frac{2}{3}$

Use a calculator to evaluate.

a. $\sin \frac{3\pi}{8}$

b. $\csc 3$

Section 4.2 p. 299; 5-12, 13-37 odd, 43-47 odd