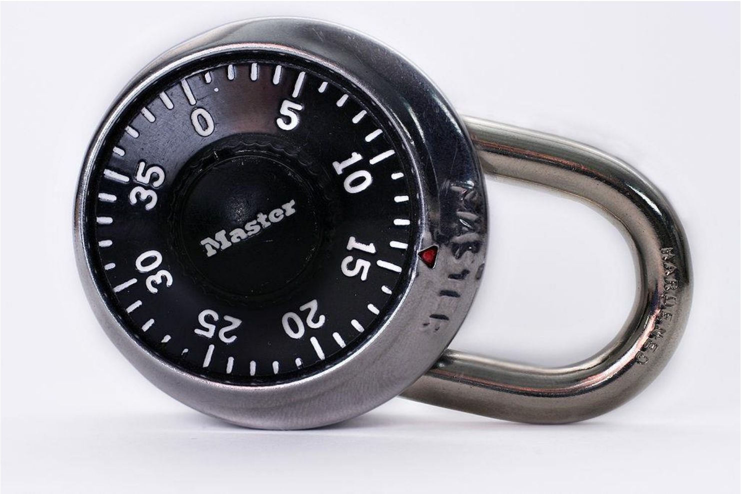
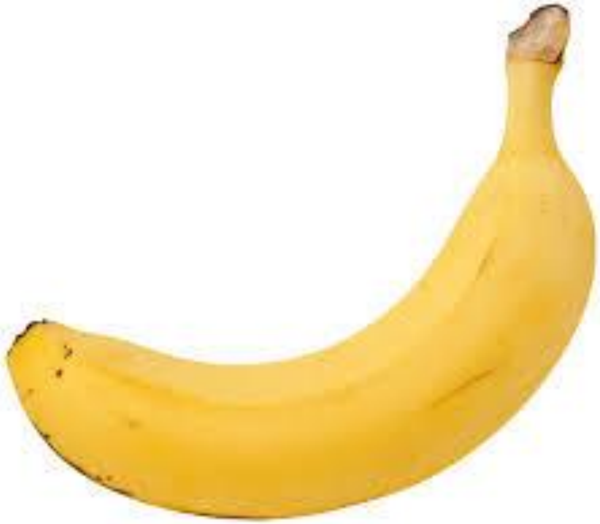
Counting Principles

Lesson 9.6





Counting Principles

- Are used to find the number of possible outcomes for a sequence of events
- Rules:
  - Fundamental Counting Rule
  - Permutation Rule
    - Of “n” elements
    - Of “n” elements taken “r” at a time
    - Distinguishable Permutations
  - Combination Rule

Fundamental Counting Principle

A combination lock has numbers from 0 to 39. It takes 3 correct numbers to unlock it. How many possible lock combinations exist?

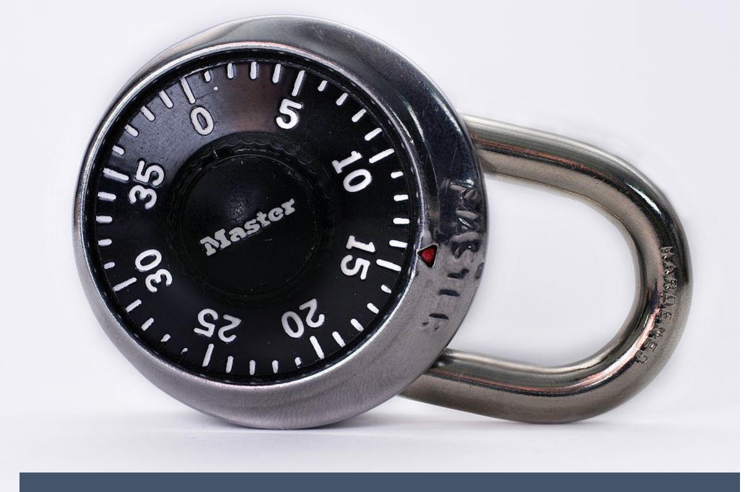
40 · 40 · 40

64,000

402 -

8 10 10 - 10 10 10 10

0or1 $8 \cdot 10^6$



In a series of events, the number of possible outcomes can be found by multiplying the possible outcomes of each individual event.

With or Without replacement?

In Douglas, Lancaster, and Sarpy counties, Alpha/Numeric plates are issued. If all letter combinations were available, how many distinct license plates could be formed?

26 26 26 10 10 10

How many batting lineups are possible in a 9 player baseball team if the pitcher can't bat in the first four spots?

$$\underbrace{\underline{8}\quad\underline{7}\quad\underline{6}\quad\underline{5}}_{-\text{pitcher}}\quad\underline{5}\quad\underline{4}\quad\underline{3}\quad\underline{2}\quad\underline{1}$$

Permutations (Order Matters!)

- Number of Permutations of "n" Elements
  - How many permutations are possible for the numbers 1, 2, 3, 4, 5, and 6? $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
  - Formula for permutations: **n!** $6!$
- Number of Permutations of "n" Elements Taken "r" at a Time
  - Twelve runners are running in a race. In how many ways can these athletes come in first, second, and third?
  - Formula: ${}_{n}P_{r} = \dfrac{n!}{(n-r)!}$

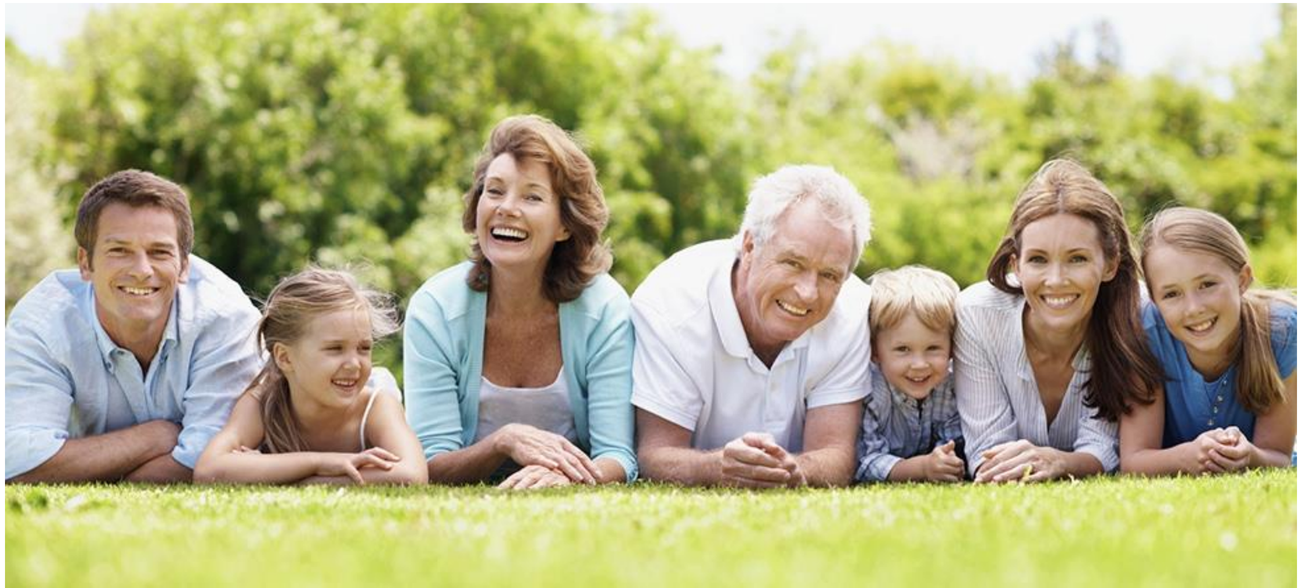
$\underline{12}\ \underline{11}\ \underline{10}$ $\quad \dfrac{12!}{9!}$

${}_{12}P_{3}$

1320

In how many ways can seven people line up in a row?

7 6 5 4 3 2 1



Distinguishable Permutations

If a set of "n" objects has n_1 of one kind of object, n_2 of a second kind, and so on, then the number of distinguishable permutations of the n objects is:

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!\,r!}$$

Example: In how many distinguishable ways can the word "banana" be written?

1 3 2

$$\frac{6!}{1!\,3!\,2!} \quad \frac{6\cdot 5\cdot 4\cdot \cancel{3!}}{2\cdot 1\cdot \cancel{3!}} = 60$$

How about PAPILLION?

$$\frac{9!}{2!\,1!\,2!\,2!\,1!\,1!} \quad \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot \cancel{4}\cdot 3\cdot \cancel{2}\cdot 1}{\cancel{2\cdot 2\cdot 2}} \quad 45,360$$

Combination Rule (Order Does Not Matter!)

You are forming a 12-member swim team (6 girls and 6 boys) from 10 girls and 15 boys. How many different teams are possible?

$\frac{10!}{6!\,4!}$ $\frac{15!}{9!\,6!}$

${}_{10}C_6$ ${}_{15}C_6$

girls boys

$210 \times 5005 = 1{,}051{,}050$

The number of selections of "n" objects without regard for order using "r" objects at a time is:

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Permutation vs. Combination

Decide whether order maters or not.

Look for key words

Permutation	Combination
Order	Selection
Arrange	Chosen
Awards	Without regard to order
Seating	
Rank	

Section 9-6 p.659; 19, 23, 27, 41, 45, 46, 59-67 odd, 75-81 odd