Operations with Matrices

Section 8.2

Equality of Matrices

Two matrices are equal if they have the same order and each corresponding entry is equal.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 7 & 13 \end{bmatrix}$$

Solve for a_{11}, a_{12}, a_{21}, and a_{22}

-3 -2 7 13

Addition of Matrices

If two matrices are of the same order, they can be added together to form a new matrix.

If $A = \begin{bmatrix} 1 & 5 & -3 \\ 0 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & -1 & 3 \end{bmatrix}$ find A+B. $= \begin{bmatrix} 1 & 6 & 2 \\ 2 & 1 & 10 \end{bmatrix}$

2 x 3 2 x 3

$$\begin{bmatrix} 4 \\ -6 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

3x1 3x1 3 x 1

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 7 & 5 \end{bmatrix} =$$ not possible

Scalar Multiplication and Matrix Subtraction

You can multiply every element of a matrix by a number called a scalar.

If $A = \begin{bmatrix} 4 & -1 \\ 0 & 4 \\ -3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 1 & 7 \end{bmatrix}$ find the following:

a. -3A b. 2B c. -3A + 2B

$$\begin{bmatrix} -12 & 3 \\ 0 & -12 \\ 9 & -24 \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ -2 & 6 \\ 2 & 14 \end{bmatrix} = \begin{bmatrix} -12 & 11 \\ -2 & -6 \\ 11 & -10 \end{bmatrix}$$

Properties of Matrix Addition and Scalar Multiplication

$A + B = B + A$	Commutative Property of Matrix Addition
$A + (B + C) = (A + B) + C$	Associative Property of Matrix Addition
$(cd)A = c(dA)$	Associative Property of Scalar Multiplication
$1A = A$	Scalar Identity Property
$c(A+B) = cA + cB$	Distributive Property
$(c + d)A = cA + dA$	Distributive Property

Simplify

$$\frac{1}{2}\left(\begin{bmatrix} 8 & 11 \\ -15 & 6 \end{bmatrix} - \begin{bmatrix} 10 & 3 \\ 5 & -6 \end{bmatrix}\right)$$

$$\frac{1}{2}\left(\begin{bmatrix} -2 & 8 \\ -20 & 12 \end{bmatrix}\right)$$

$$\begin{bmatrix} -1 & 4 \\ -10 & 6 \end{bmatrix}$$

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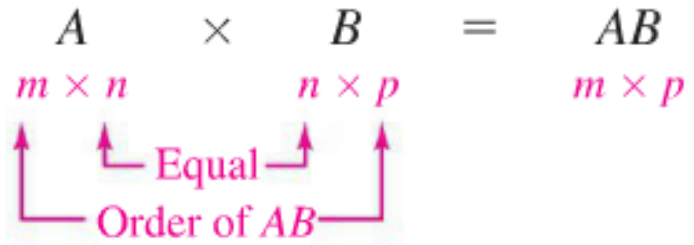
Matrix Multiplication

Section 8.2 Day 2



Find AB if $A = \begin{bmatrix} 0 & 5 & -3 \\ 6 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 0 \\ 1 & 2 & 2 \\ 11 & -6 & 9 \end{bmatrix}$

2 x 3 3 x 3

2 x 3

3 x 3 2 x 3

Find the product and resulting dimension of the following.

2x3

a. $\begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 0 & 3 & -2 \\ 2 & 1 & 4 \end{bmatrix}$

2x2 2x3

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\begin{bmatrix} 0\cdot0+1\cdot2 & 0\cdot3+1\cdot1 & 0\cdot(-2)+1\cdot4 \\ 7\cdot0+2\cdot2 & 7\cdot3+2\cdot1 & 7(-2)+2(4) \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 4 & 23 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 \\ -4 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = a_{11}\begin{bmatrix} 5 \\ -12 \\ 0 \end{bmatrix}$

3x2 2x1

$2(3)+1(-1)$

$(-4)3+0(-1)$

$3(1)+3(-1)$

c. $\begin{bmatrix} 0 & 2 & 4 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 5 \\ 3 & -2 & 4 \end{bmatrix}$ not possible

2x3 2x3

Find A^2

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 19 & -12 \\ 18 & -5 \end{bmatrix}$$

2×2 2×2 2×2

Given $\begin{cases} -x_1 + 3x_2 = 3 \\ x_1 - x_2 = 3 \end{cases}$

coefficient variables answer

$\begin{bmatrix} -1 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

- Write the system as a matrix equation, AX = B.
- Use Gauss-Jordan elimination on the augmented matrix $[A : B]$ to solve for matrix X.

$-1(R1) \rightarrow \begin{bmatrix} -1 & 3 & 3 \\ 1 & -1 & 3 \end{bmatrix} \xrightarrow{R1+R2} \begin{bmatrix} 1 & -3 & -3 \\ 0 & 2 & 6 \end{bmatrix} \xrightarrow{+3(R2)+R1} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{\frac{1}{2}R3}$

$\begin{bmatrix} 1 & 0 & \\ 0 & 1 & \end{bmatrix}$

$X_1 = 6$

$X_2 = 3$

A farmer raises two crops which are shipped to 3 markets. The profit per unit for crop 1 is $3.75 and the profit per unit for crop 2 is $7. Find the profit for each market.

	Market 1	Market 2	Market 3
Crop 1	100	75	75
Crop 2	125	150	100

$$\begin{bmatrix} 3.75 & 7 \end{bmatrix} \begin{bmatrix} 100 & 75 & 75 \\ 125 & 150 & 100 \end{bmatrix}$$

$1 \times 2 \quad 2 \times 3$

$$\begin{bmatrix} 375+ & \cdot & \cdot \\ 875 & & \end{bmatrix}$$

$$\begin{bmatrix} 1250 & 1331.25 & 981.25 \end{bmatrix}$$

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