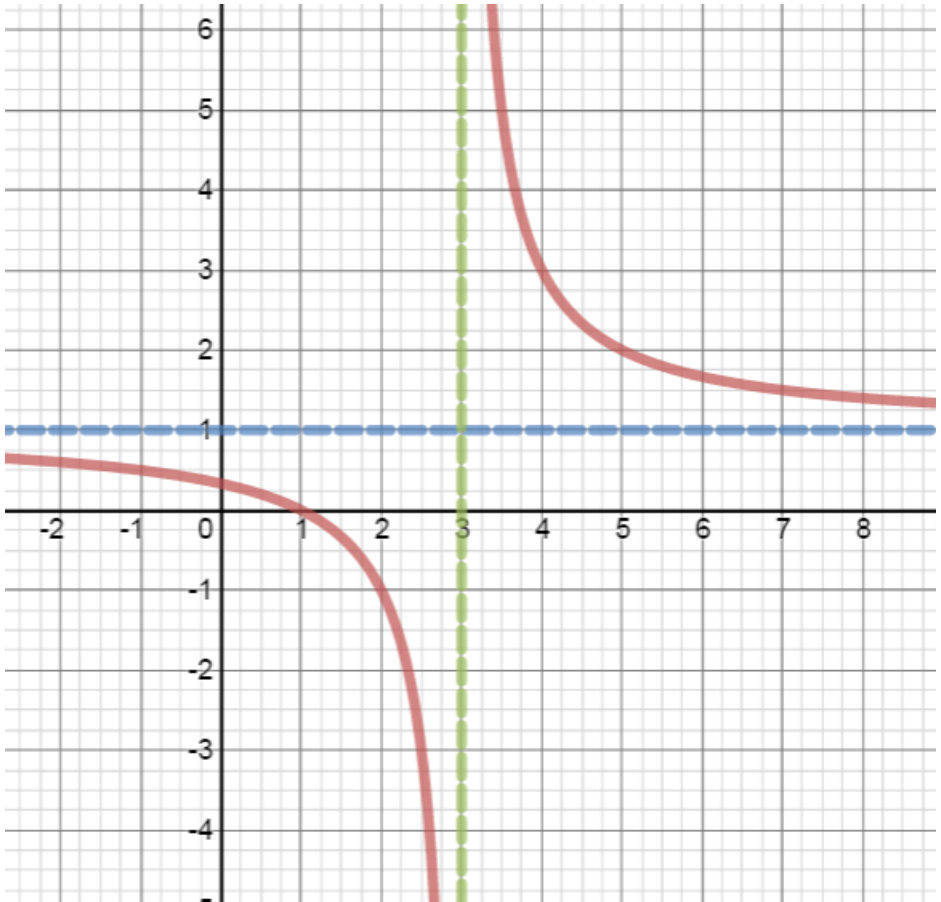
Rational Functions

Section 2.6

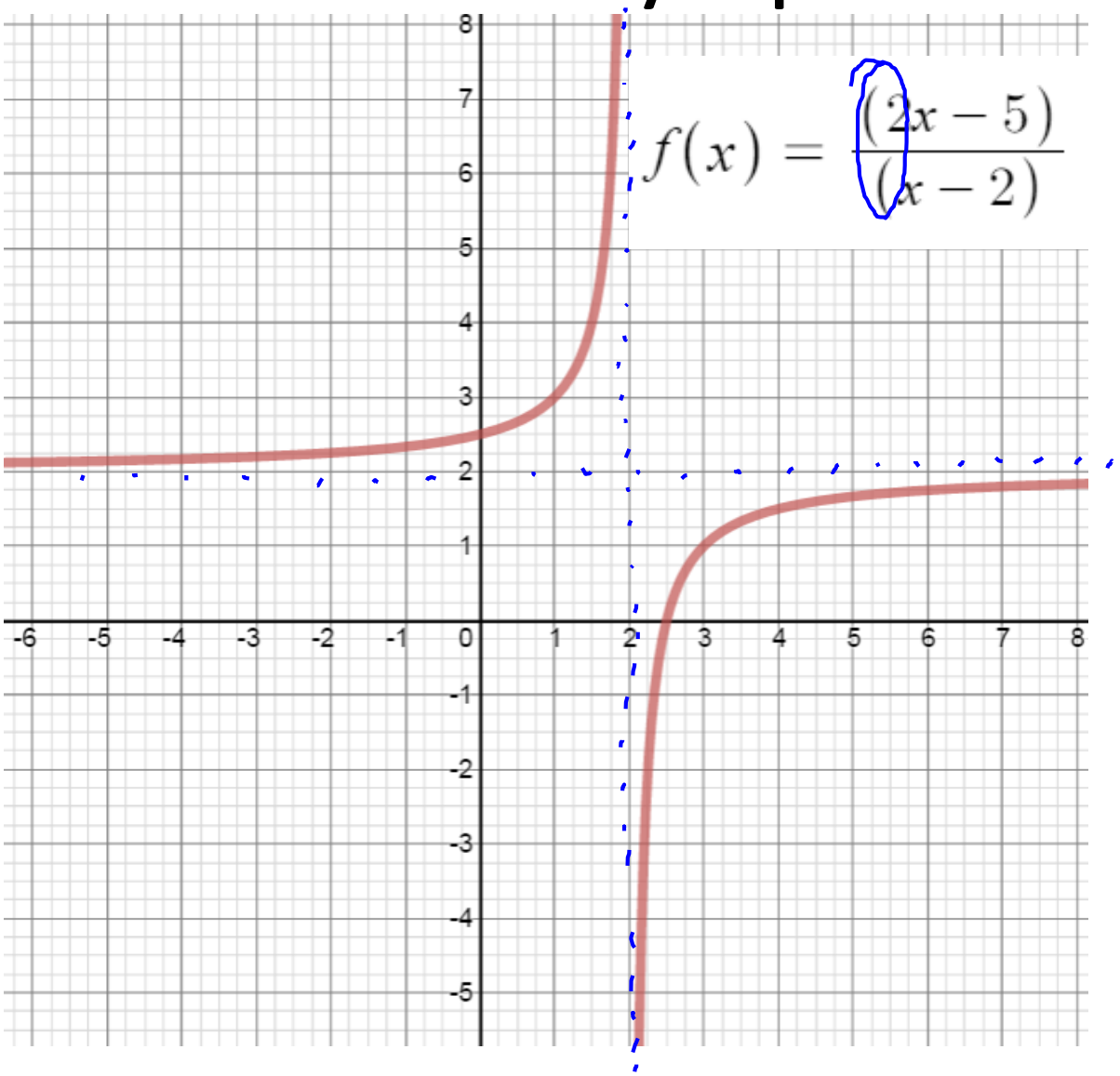


- A rational function is of the form f(x) – $\frac{N(x)}{D(x)}$where N and D are both polynomials. The domain of f is all x's such that d(x) $\neq 0$.
- N(x) and D(x) can't have common factors (simplify first!)

Horizontal and Vertical Asymptotes

Vertical – occur where D(x) = 0

Horizontal – the graph may cross the horizontal asymptote provided it approaches the line as it goes to infinity.



Rules of Asymptotes

The graph of f has one horizontal asymptote or no horizontal asymptotes depending on the degree of N and D.

If the degree of N < the degree of D then the x-axis (y = 0) is the horizontal asymptote.

If the degree of N and D are the same then y = the ratio of the leading coefficients $\left(\frac{a_n}{a_d}\right)$.

If the degree of N > the degree of D then there's no horizontal asymptote. (Slant asymptote)

A Plan of Attack

1. Simplify f if possible
2. Find and sketch vertical asymptotes.
3. Find and sketch horizontal or slant asymptotes
4. Find x and y intercepts if possible
5. Plot at least two points per branch (x-y chart)
6. Connect points with smooth curve

Graph $f(x) = \dfrac{2x^2}{x^2-4}$

$\frac{2}{-3}$ $\frac{18}{45}$ $\frac{32}{12}$ $\dfrac{2x^2}{(x+2)(x-2)}$

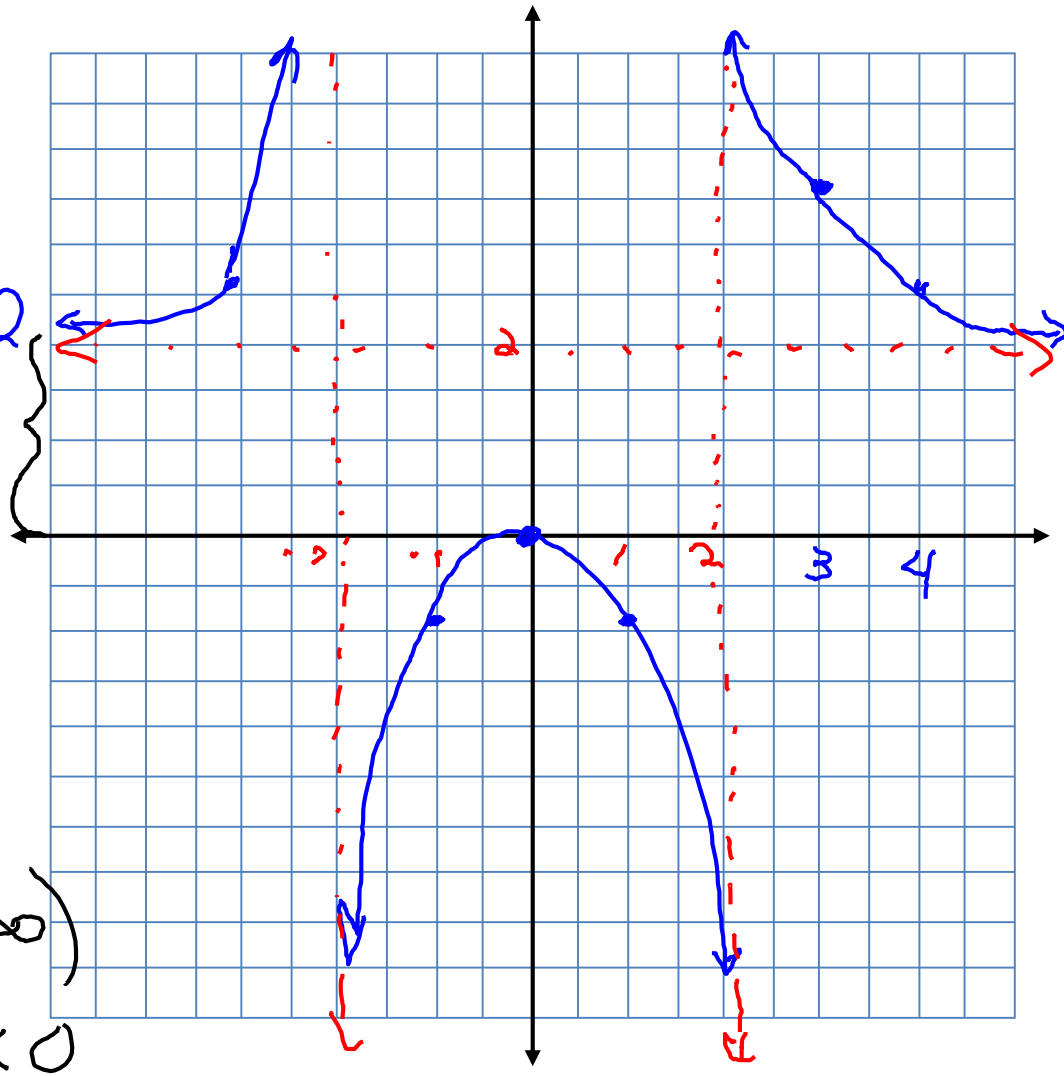
Vertical – $x=2,\ x=-2$

Horizontal – $y=2$

Domain – $\mathbb{R}$ s.t. $x \neq 2$

$(-\infty,-2)(-2,2)(2,\infty)$

x	y
0	0
1	-2/3
-1	-2/3
3	3 3/5



Range - $(-\infty, 0]\cup(2,\infty)$

$y \geq 2$ or $y < 0$

top > bottom slant

$\frac{a}{b}$ top = bottom

y = 0 top < bottom

Graph $f(x) = \frac{x^2+x-2}{x^2-x-6}$

$\frac{(x+2)(x-1)}{(x-3)(x+2)}$

hole $x = -2$, $\left(-2, \frac{3}{5}\right)$

Vertical – $x = 3$

Horizontal – $y = 1$

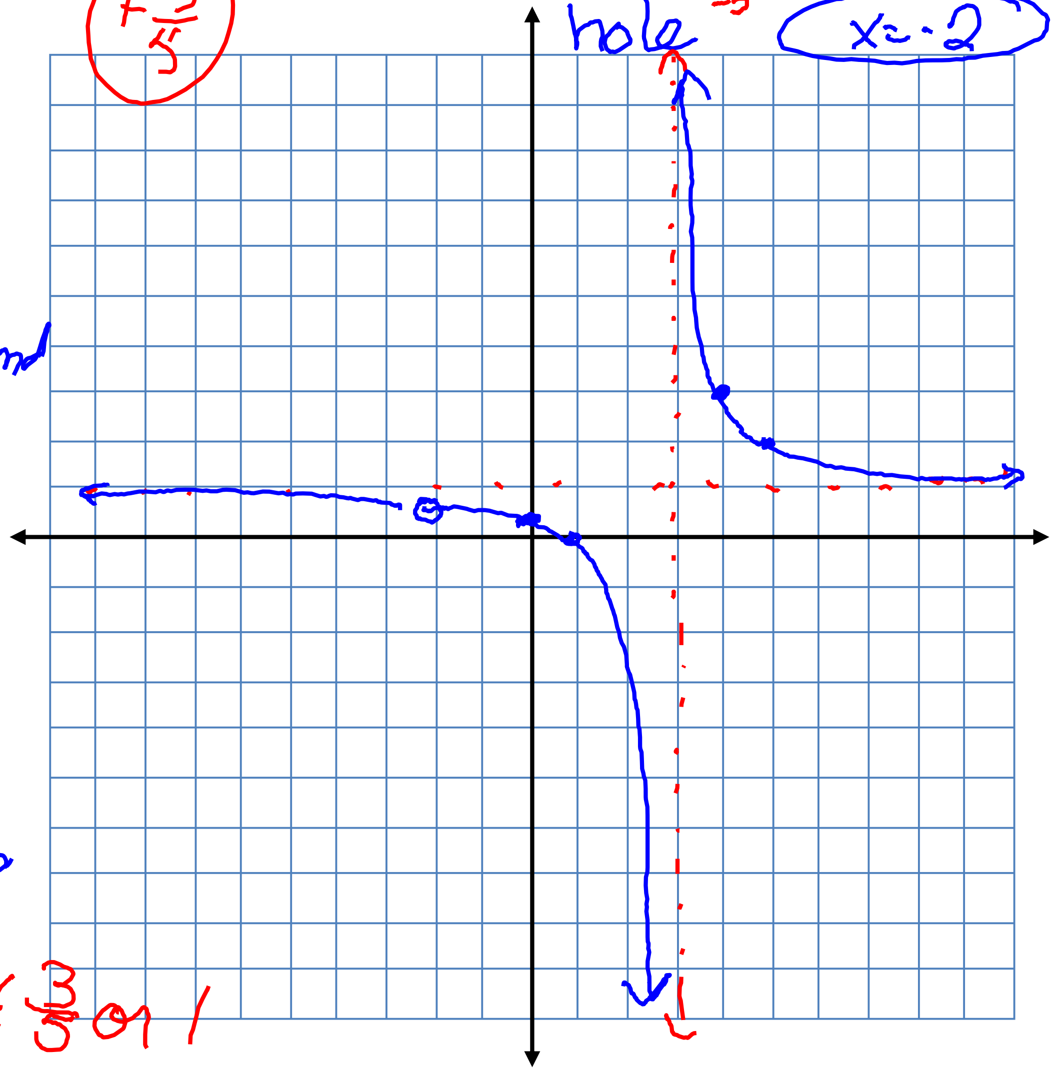
Domain – ℝ s.t. $x \neq 3$ and $x \neq -2$

x	y
0	$\frac{1}{3}$
1	0
4	3
5	2

$\frac{16+4-2}{16-4-6} = \frac{18}{6}$

$\frac{25+5-2}{25-5-6} = \frac{28}{14}$

Range - ℝ s.t. $y \neq \frac{3}{5}$ or 1



Slant Asymptote

Occurs when the degree of N is exactly one more than the degree of D.

Find it by using long division (or synthetic if possible).

Slant asymptote function is the quotient of the division (don’t worry about the remainder).

0	0
1	0
-2	-6

Graph $f(x) = \dfrac{x^2 - x}{x+1}$

Vertical – $x = -1$

Domain – $\mathbb{R}$ s.t. $x \neq -1$

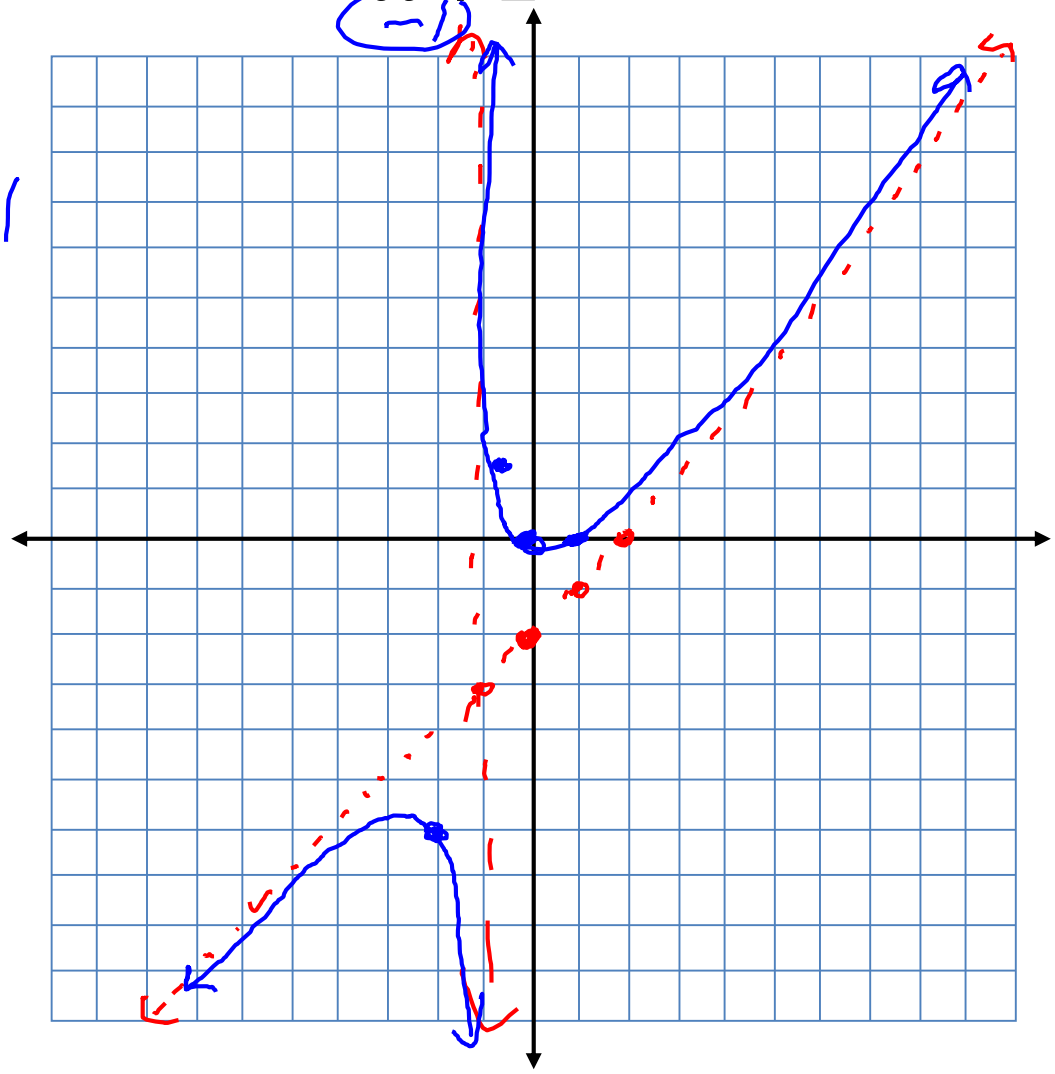
Slant Asympt. -

$-1 \;|\; 1 \;\; -1 \;\; 0$

$\downarrow \;\; -1 \;\; 2$

$1 \;\; -2 \;|\; 2$

$y = x - 2$



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9-15 odd, 17, 26, 27, 29, 33, 57, 60