Trigonometric Form of Complex Numbers

Section 6.5

The Complex Plane

$z = a + bi$

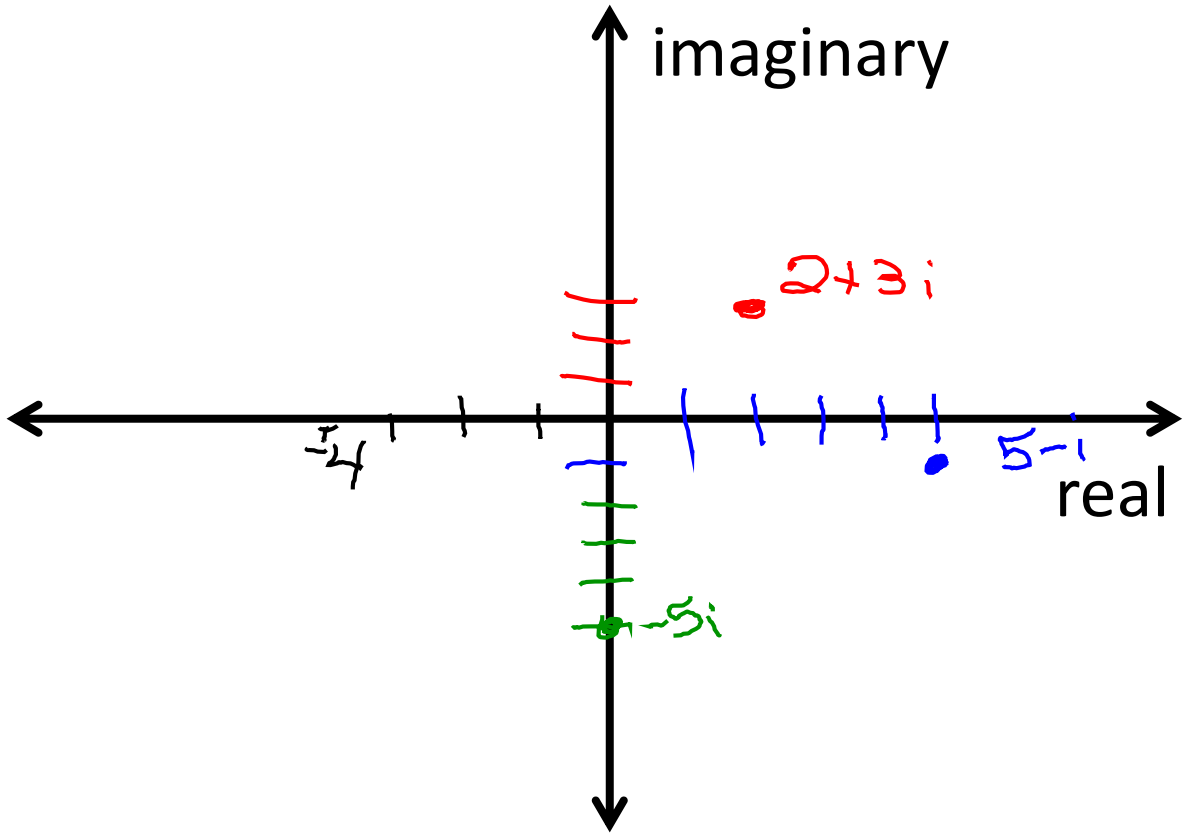
Graph

$2 + 3i$

$5 - i$

$-5i$

-4

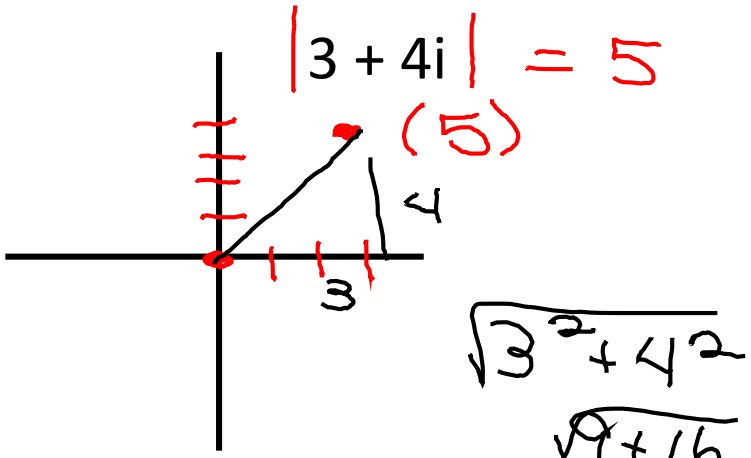


Absolute Value

What is |-3| and |5|? What do these values represent?

Absolute Value of a Complex Number

The length of the line segment from the origin to the point.



Find $|-5 + 2i|$.

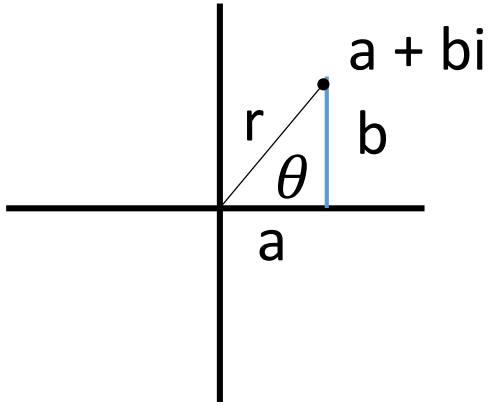
$a + bi$

$|a+bi| = \sqrt{(-5)^2 + (2)^2}$

$\sqrt{25 + 4}$

$\sqrt{29}$

Trig Form of a Complex Number



$\cos\theta = \frac{a}{r}$

$r\cos\theta = a$

a. How do we determine θ? $\tan\theta = \frac{b}{a}$

b. How are a, θ, and r related? $a = r\cos\theta$

c. How are b, θ, and r related? $b = r\sin\theta$

$z = r \bullet \cos\theta + i \bullet r \bullet \sin\theta \quad \text{or} \quad z = r(\cos\theta + i\sin\theta)$

$3+6i \quad \sqrt{45}\cos\theta + i\sqrt{45}\sin\theta$

Given the complex number -5 + 6i, write it in trig form.

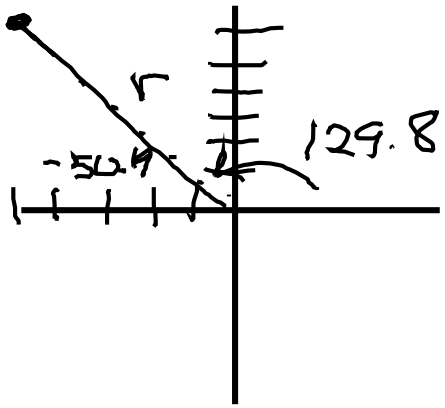
$z = r(\cos\theta + i\sin\theta)$

$\sqrt{61}(\cos 129.8 + i\sin 129.8)$

$r = \sqrt{25+36}$

$= \sqrt{61}$

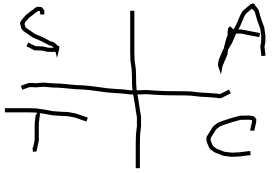
$\tan\theta = \frac{6}{-5}$



Write $z = 3(\cos 315° + i \sin 315°)$

$3\left(\frac{\sqrt{2}}{2} + i\,\frac{-\sqrt{2}}{2}\right)$

$\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$



Product/Quotient of Complex Numbers in Trig Form

$\frac{3+2i}{5-7i}$

Given $z_1 = r_1 (\cos \theta_1 + i \sin\theta_1)$ $z_2 = r_2 (\cos \theta_2 + i \sin\theta_2)$

Product

$$z_1 z_2 = r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)]$$

Quotient

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos (\theta_1 - \theta_2) + i \sin (\theta_1 - \theta_2)]$$ rcis ⦞

Note: The angle must be between 0° and 360°.

Find $(4 \bullet \text{cis } 120°) \bullet (6 \bullet \text{cis } 315°)$

$(4\cos 120 + i \sin 120)(6\cos 315 + i \sin 315)$

$24(\cos(435) + i \sin(435))$

$24(\cos 75° + i \sin 75°)$

$$\begin{array}{r} 435 \\ -360 \\ \hline 75 \end{array}$$

Find $\frac{15\ cis\ 240°}{3\ cis\ 135°}$

$5(\cos 105 + i \sin 105)$

$5\ cis\ 105°$

Section 6-5 p. 448; 5-6, 11, 17, 25-26, 31, 34, 45-54 x3s, 57, 61