Dot Products and Work

Section 6.4

Definition of the Dot Product

The dot product of **u** = $\langle u_1, u_2 \rangle$ and **v** = $\langle v_1, v_2 \rangle$ is

u • **v** = $u_1v_1 + u_2v_2$

Note the dot product results in a scalar, not a vector.

If **v** = -2**i** + 5**j** and **w** = 4i + j, find **v** • **w**.

$-2(4)+(5)(1) = -8+5 = -3$

If **u** = $\langle 2, 1 \rangle$ and **v** = $\langle -1, 0 \rangle$, find **u** • **v**.

$-2+0 = -2$

Properties of the Dot Product (p. 431)

Properties of the Dot Product

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in the plane or in space and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{0} \cdot \mathbf{v} = 0$
3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

$v = \langle a, b \rangle \cdot \langle a, b \rangle$

$a^2 + b^2$

Use the Dot Product to Find the Angle Between Two Vectors

- If θ is the angle between two non-zero vectors **u** and **v**, then:

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

There are technically and infinite number of angles between two vectors. This finds the shortest angle between them. Also if the angle between the vectors is 0° or 180°, then the vectors are parallel.

Find the angle between **v** and **w** if **v** = -2i + 5j and **w** = 4i + j.

$\cos\theta = \frac{-3}{(\sqrt{29}\cdot\sqrt{17})}$

$\cos\theta = -.135$

$\theta = 97.77$

$\frac{-3}{\sqrt{29}\cdot\sqrt{17}}$

$-2(4)+5(1)$

$\|v\| = \sqrt{(-2)^2+5^2}$

$\sqrt{29}$

$\|w\| = \sqrt{4^2+1^2}$

$\sqrt{17}$

Orthogonal

- Orthogonal Vectors are perpendicular to each other – they form a right angle.
- $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$ $= 0$
- If the angle is 90°, what does this mean with regard to the formula?
- If the dot product is 0, the vectors are orthogonal.

Are vectors **u** = $\langle -2, 3 \rangle$ and **v** = $\langle 6, 4 \rangle$ orthogonal?

$-2(6) + 3(4)$

$-12 + 12 = 0$ yes

Work

- Work = F x D (Physics Formula)
- F = the component force in the direction of the motion
- D = Distance

A tractor pulls a log 800 meters, and the tension n in the cable connecting the tractor and log is approximately 1600 kg. The direction of the force is 35° above the horizontal. Approximate the work done when pulling the log.

$W = FD$

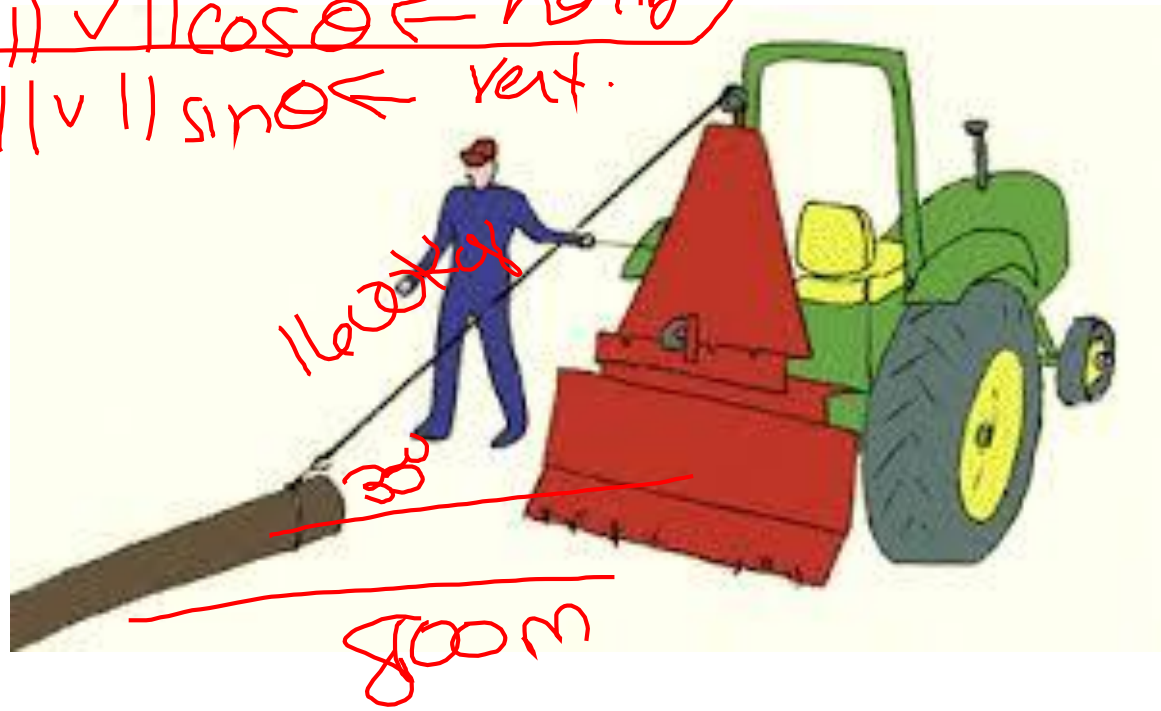
$F = 1600 \cos 35°$

$W = 1600 \cos 35 \, (800)$

$W = 1048514.62 \text{ kgm}$

$||v|| \cos\theta \leftarrow$ horiz.

$||v|| \sin\theta \leftarrow$ vert.



Section 6.4 p. 437; 7-37 x 3’s, 53, 57, 58, 79-81