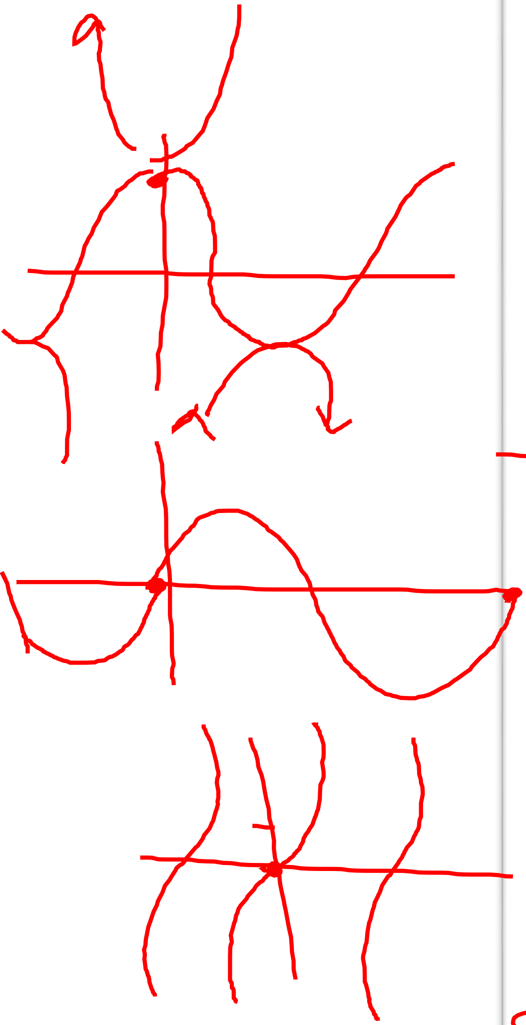
Using Fundamental Identities

Section 5.1



Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

$\cos^2 u$ $\cos^2 u$ $\cos^2 u$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

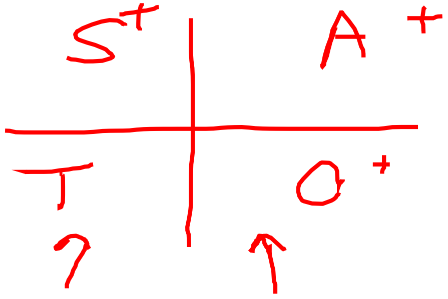
odd $\sin(-u) = -\sin u$ even $\cos(-u) = \cos u$ odd $\tan(-u) = -\tan u$

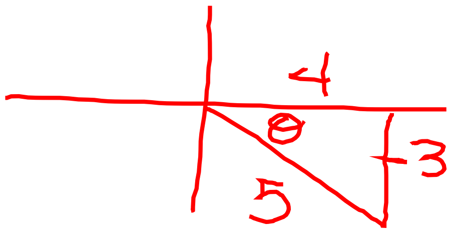
odd $\csc(-u) = -\csc u$ even $\sec(-u) = \sec u$ odd $\cot(-u) = -\cot u$

If $\csc u = -\frac{5}{3}$ and cos u > 0, find the value of the other five trig functions.

hyp, opp

Q IV





$\sin u = -\frac{3}{5}$

$\sin^2 u + \cos^2 u = 1$

$\left(-\frac{3}{5}\right)^2 + \cos^2 u = 1$

$\frac{9}{25} + \cos^2 u = \frac{25}{25}$

$-\frac{9}{25} \qquad -\frac{9}{25}$

$\sqrt{\cos^2 u} = \sqrt{\frac{16}{25}}$

$\cos u = 4/5$

$\tan u = \frac{\sin u}{\cos u}$

$\tan u = \frac{-3/5}{4/5}$

$\tan u = -\frac{3}{5} \cdot \frac{5}{4}$

$\tan u = -3/4$

$\cot u = -4/3$

$\sec u = 5/4$

Simplify $\sin^2 x \sec x - \sec x$

$\sec x(\sin^2 x - 1)$

$\sec x(-\cos^2 x)$

$\frac{1}{\cancel{\cos x}}(-\cos^{\cancel{2}} x)$

$-\cos x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x - 1 = -\cos^2 x$

Simplify tan x sin x + cos x

$\frac{2}{4} \cdot 2$

$\frac{2 \cdot 2}{4}$

$\frac{\sin x}{\cos x} \sin x + \cos x$

$\frac{\sin^2 x}{\cos x} + \cos x \frac{(\cos x)}{\cos x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x}$

$\frac{1}{\cos x}$

$\sec x$

Simplify $\dfrac{\sec t}{\tan t} - \dfrac{\tan t}{1+\sec t}$

$$\frac{\cancel{\cos t}}{\sin t}\cdot\frac{\frac{1}{\cancel{\cos t}}}{\frac{\sin t}{\cancel{\cos t}}} \qquad \frac{\cancel{\cos t}}{\cancel{\sin t}} \qquad - \frac{\frac{\sin t}{\cancel{\cos t}}}{\frac{\cos t}{\cos t} + \frac{1}{\cos t}}\cdot\frac{\cancel{\cos t}}{\cos t + 1} \qquad \frac{\cos t + 1}{\cancel{\cos t}} \qquad \frac{\cancel{\cos t}}{\cancel{\cos t}+1}$$

$$\frac{\cos t + 1}{\cos t + 1}\cdot\frac{1}{\sin t} - \frac{\sin t}{\cos t + 1}\cdot\frac{\sin t}{\sin t}$$

$$\frac{\cos t + 1 - \sin^2 t}{\sin t(\cos t + 1)} \qquad \frac{\cos t + \cos^2 t}{\sin t(\cos t + 1)} \Rightarrow \frac{\cos t\,(1+\cancel{\cos t})}{\sin t\,(\cancel{\cos t + 1})}$$

$\cot t$

Factor the following trig expressions

$x^2 - x - 2$
$(x+1)(x-2)$

$y^2 - 1$
$(y+1)(y-1)$

$x^2 - 3x - 10$
$(x+2)(x-5)$

$\cancel{x^2 - y - 3}$

a. $\cos^2 x - 1$

$(\cos x + 1)(\cos x - 1)$

b. $\sin^2 u - 3 \sin u - 10$

$(\sin u - 5)(\sin u + 2)$

c. $\cancel{\sec^2} t - \tan t - 3$

$\underline{1 + \tan^2 t} - \tan t - 3$
$\tan^2 t - \tan t - 2$
$(\tan t + 1)(\tan t - 2)$

$\sin^2 x + \cos^2 x = 1$

Rewrite $\dfrac{1}{\boxed{\sec x - 1}}$ so that it is not a fraction

$$\frac{1}{(\sec x - 1)} \frac{(\sec x + 1)}{(\sec x + 1)}$$

$$\frac{\sec x + 1}{\sec^2 x - 1} \Rightarrow \frac{\sec x + 1}{\tan^2 x}$$

$$\frac{1}{x-1}$$

$$\frac{3+1}{4} = \frac{3}{4} + \frac{1}{4}$$

$$\cancel{\frac{4}{3+1} = \frac{4}{3} + \frac{4}{1}}$$

$$\frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x}$$

$$\downarrow$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cancel{\cos x}} \qquad \frac{\cos x}{\sin^2 x} + \cot^2 x$$

$$\cancel{\frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{\cos x}{\sin^2 x} + \cot^2 x$$

$$\frac{\cos x}{\sin x} \frac{1}{\sin x} + \cot^2 x$$

$$\cot x \csc x + \cot^2 x$$

Use the substitution x = 3sin u, 0 < u < π/2 to express $\sqrt{9 - x^2}$ as a function of u.

$\sqrt{9-(3\sin u)^2}$

$\sqrt{\frac{9}{9} - \frac{9}{9}\sin^2 u}$

$\sqrt{9(1-\sin^2 u)}$

$\sqrt{9\cos^2 u}$

$3\cos u$

$\sqrt{25-3^2}$

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