Multiple-Angle and Product-to-Sum Formulas

Section 5.5

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \frac{2 \tan u}{1 - tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= 2 \cos^2 u - 1$

$= 1 - 2 \sin^2 u$

Solve $3\sin^2 x + \cos 2x - 2 = 0$

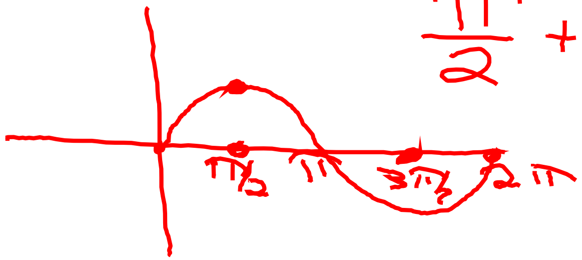
$3\sin^2 x + 1 - 2\sin^2 x - 2 = 0$

$\sin^2 x - 1 = 0$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$

$\frac{\pi}{2} + \pi n$



$\cos 2u = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

Rewrite the expression using a double angle formula.

$4 \sin x \cos x$ $= 2 \sin 2x$

$2(\sin 2u) = 4 \sin u \cos u$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Rewrite $\sin^2 x \cos^2 x$ in terms of first powers of the cosines of multiple angles.

$\frac{1-\cos 2x}{2} \quad \frac{1+\cos 2x}{2}$

$\frac{1+\cos 2x - \cos 2x - \cos^2 2x}{4}$

$\frac{1-\cos^2 2x}{4}$

$\frac{\frac{2}{2}1 - \frac{1+\cos 4x}{2}}{4} \Rightarrow \frac{\frac{2-1-\cos 4x}{2}}{4}$

$\frac{\frac{1-\cos 4x}{2}}{4}$

$\frac{1-\cos 4x}{8}$

Power-Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

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