$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

5.1 – 5.2 Review

Name Key

Simplify

1. $\csc t - \cos t \cot t$

$\csc t - \cos t \frac{\cos t}{\sin t}$

$\frac{\sin t}{\sin t}\csc t - \frac{\cos^2 t}{\sin t}$

$\frac{\sin t \csc t - \cos^2 t}{\sin t} \Rightarrow \frac{1-\cos^2 t}{\sin t} \Rightarrow \frac{\sin^2 t}{\sin t} \Rightarrow \sin t$

2. $\frac{1}{1+\sin\phi} + \frac{1}{1-\sin\phi}$ $\quad \left(\frac{1-\sin\phi}{1-\sin\phi}\right)\left(\frac{1+\sin\phi}{1+\sin\phi}\right)$

$\frac{1-\sin\phi + 1+\sin\phi}{(1+\sin\phi)(1-\sin\phi)} \Rightarrow \frac{2}{1-\sin^2\phi} \Rightarrow \frac{2}{\cos^2\phi} \Rightarrow 2\sec^2\phi$

3. $\frac{\cos^2\phi}{1-\sin\phi} \Rightarrow \frac{1-\sin^2\phi}{1-\sin\phi} \Rightarrow \frac{(1-\sin\phi)(1+\sin\phi)}{1-\sin\phi}$

$(1+\sin\phi)$

4. $(\tan^2 x)\csc^2 x - \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x}\frac{1}{\sin^2 x} - \tan^2 x$

$\frac{1}{\cos^2 x} - \tan^2 x$

$\sec^2 x - \tan^2 x$

1

5. $\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin x} \Rightarrow \frac{\cos x}{\sin x} \Rightarrow \cot x$

6. $(\tan x + 1)^2 \cos x$ $\quad (x+1)^2 \Rightarrow (x+1)(x+1)$ $\quad x^2+2x+1$

$(\tan^2 x + 2\tan x + 1)\cos x$

$(\sec^2 x + 2\tan x + 1)\cos x$

$\left(\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x}\right)\cos x$

$\frac{\cos x}{\cos^2 x} + \frac{2\sin x \cos x}{\cos x}$

$\frac{1}{\cos x} + 2\sin x$

$\sec x + 2\sin x$

Use the values given and the fundamental trig identities to find the values of all six trig functions.

7. $\tan x = -\frac{1}{3}$ and $\cos x > 0$

S | A
T | C

QIV

$1 + \tan^2 x = \sec^2 x$

$1 + \left(-\frac{1}{3}\right)^2 = \sec^2 x$

$1 + \frac{1}{9} = \frac{9}{9} = \sec^2 x$

$\sqrt{\frac{10}{9}} = \sec^2 x$

$\frac{\sqrt{10}}{3} = \sec x$

$\tan x = \frac{\sin x}{\cos x}$

$-\frac{1}{3} = \frac{\sin x}{\frac{3\sqrt{10}}{10}}$

$\sin x = -\frac{1}{3}\cdot\frac{\sqrt{10}}{10} = -\frac{\sqrt{10}}{10}$

$\sin x = -\frac{\sqrt{10}}{10}$ $\quad \csc x = -\sqrt{10}$

$\cos x = \frac{3\sqrt{10}}{10}$ $\quad \sec x = \frac{\sqrt{10}}{3}$

$\tan x = -\frac{1}{3}$ $\quad \cot x = -\frac{3}{1}$

8. $\cos x = -\frac{1}{2}$ and $\csc x < 0$

S | A
T | C

QIII

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(-\frac{1}{2}\right)^2 = 1$

$\sin^2 x + \frac{1}{4} = \frac{3}{4}$

$\sin x = \frac{\sqrt{3}}{2}$

$\tan x = \frac{\sin x}{\cos x}$

$\tan x = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\tan x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$ $\quad \csc x = -\frac{2\sqrt{3}}{3}$

$\cos x = -\frac{1}{2}$ $\quad \sec x = -2$

$\tan x = \sqrt{3}$ $\quad \cot x = \frac{\sqrt{3}}{3}$

Verify the following using fundamental trig identities.

9. $\frac{1+\cos x}{1-\cos\theta} + \frac{1-\cos x}{1+\cos\theta} = 2\csc^2\phi$

$\frac{1+\cos\theta + 1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$

$\frac{2}{1-\cos^2\theta}$

$\frac{2}{\sin^2\theta} \Rightarrow 2\csc^2\theta$

10. $\frac{\sin^2\phi+\cos^2\phi}{\cos^2\phi\sec^2\phi} = 1$

$\frac{1}{\cos^2\phi\sec^2\phi}$

$\frac{1}{\cos^2\theta\frac{1}{\cos^2\theta}}$

$\frac{1}{1}$

11. $(\sec^2 x - 1)(\sin^2 x - 1) = -\sin^2 x$

$(\tan^2 x)(-\cos^2 x)$

$\frac{\sin^2 x}{\cos^2 x}(-\cos^2 x)$

$-\sin^2 x$

12. $\cos x(\tan^2 x + 1) = \sec x$

$\cos x(\sec^2 x)$

$\cos x \frac{1}{\cos^2 x}$

$\frac{1}{\cos x}$

$\sec x$

13. $\sec^2 x \cot x - \cot x = \tan x$

$\cot x(\sec^2 x - 1)$

$\cot x(\tan^2 x)$

$\frac{1}{\tan x}(\tan^2 x)$

$\tan x$

14. $\frac{\cos x-\cos y}{\sin x+\sin y} + \frac{\sin x-\sin y}{\cos x+\cos y} = 0$ $\quad \cos x+\cos y \quad \sin x+\sin y$

$\frac{(\cos x+\cos y)(\cos x-\cos y) + (\sin x-\sin y)(\sin x+\sin y)}{(\cos x+\cos y)(\sin x+\sin y)}$

$\frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\cos x+\cos y)(\sin x+\sin y)}$

$\frac{\cos^2 x + \sin^2 x - \cos^2 y - \sin^2 y}{(\cos x+\cos y)(\sin x+\sin y)}$

$\frac{1 - 1(\cos^2 y + \sin^2 y)}{(\cos x+\cos y)(\sin x+\sin y)}$

$\frac{1-1}{(\cos x+\cos y)(\sin x+\sin y)}$

0

Factor the following.

15. $\sec^2 x + 3\tan x + 1$

x^2+3x+2 $\quad (x+2)(x+1)$

$\tan^2 x + 1 + 3\tan x + 1$

$\tan^2 x + 3\tan x + 2$

$(\tan x + 2)(\tan x + 1)$

16. $\sec^3 x - \sec^2 x - \sec x + 1$

$\sec^2 x(\sec x - 1) \quad -1(\sec x - 1)$

$(\sec x - 1)(\sec^2 x - 1)$

$(\sec x - 1)(\sec x + 1)(\sec x - 1)$ or

$(\sec x + 1)(\sec x - 1)^2$