Trigonometric Functions of Any Angle

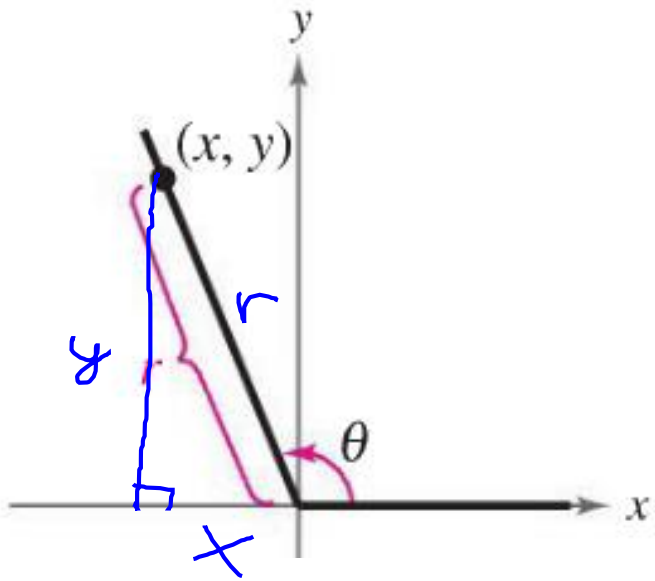
Section 4.4

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

$$\sec\theta = \frac{r}{x}, \quad x \neq 0 \qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$$



If (7, -4) is a point on the terminal side of θ, find the sine, cosine, and tangent of θ.

$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{65}} = \frac{-4\sqrt{65}}{65}$ $\sqrt{49+16} = r$

$\sqrt{65} = r$

$\csc\theta = \frac{\sqrt{65}}{-4}$

$\cos\theta = \frac{x}{r} = \frac{7}{\sqrt{65}} = \frac{7\sqrt{65}}{65}$

$\sec\theta = \frac{\sqrt{65}}{7}$

$\tan\theta = \frac{y}{x} = \frac{-4}{7}$

$\cot\theta = \frac{-7}{4}$

$(2, \sqrt{5})$

Given $\cos\theta = \frac{2}{3}$ and $\tan\theta > 0$, find $\csc\theta$.

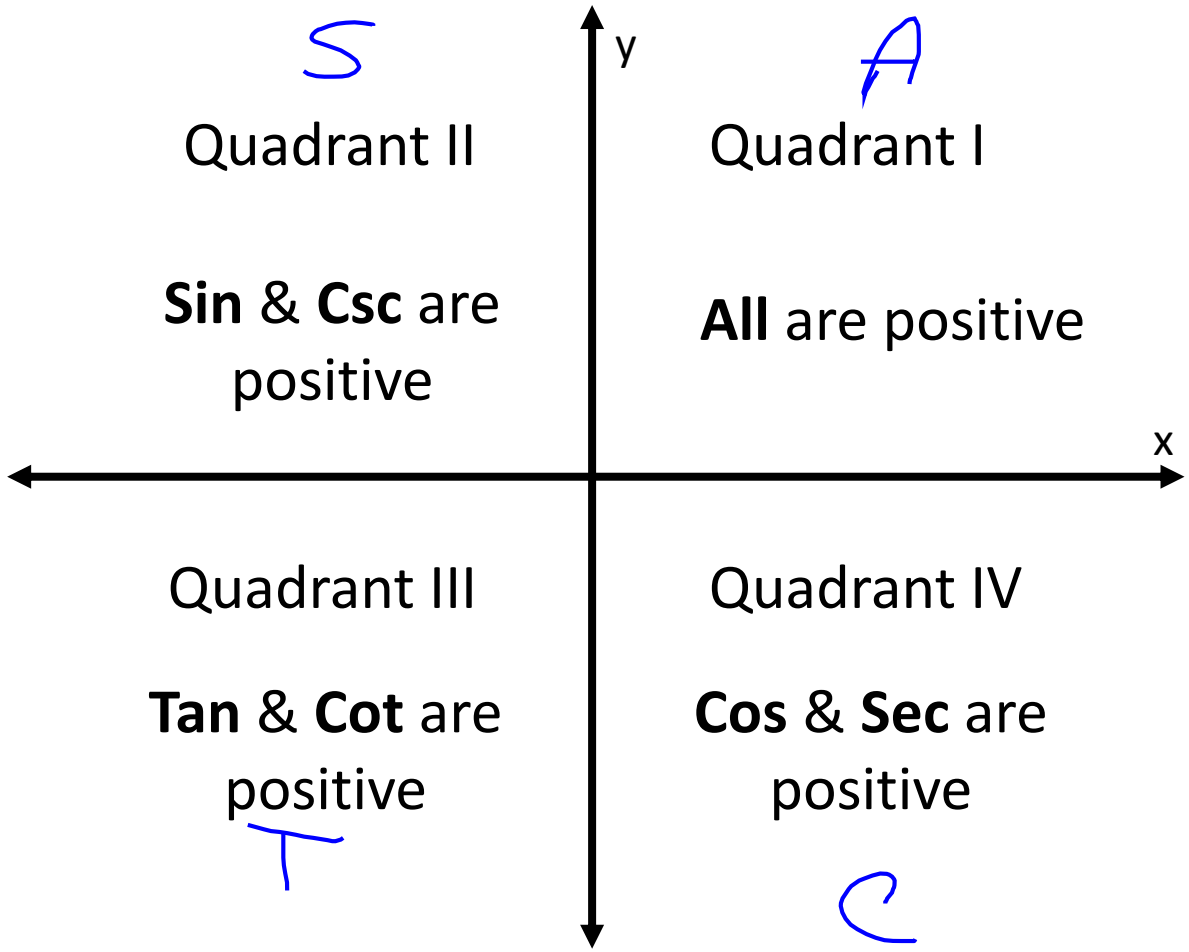
$\cos\theta = \frac{x}{r}$ $\quad \sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$(3)^2 \quad (\sqrt{2^2 + y^2})^2$

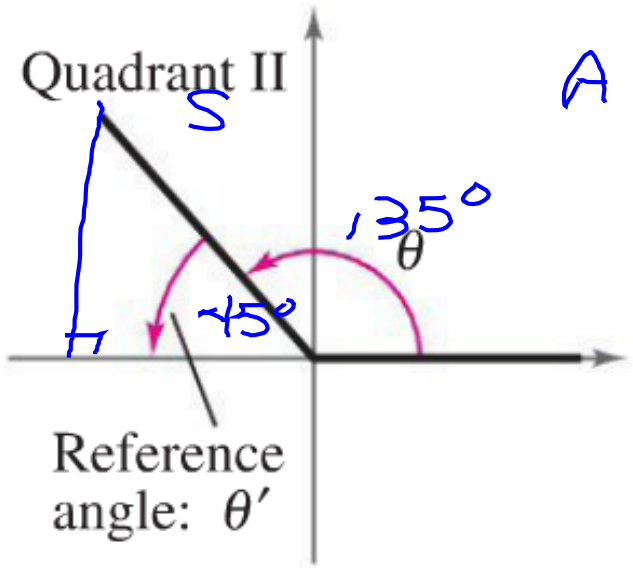
$9 = 4 + y^2$

$5 = y^2$

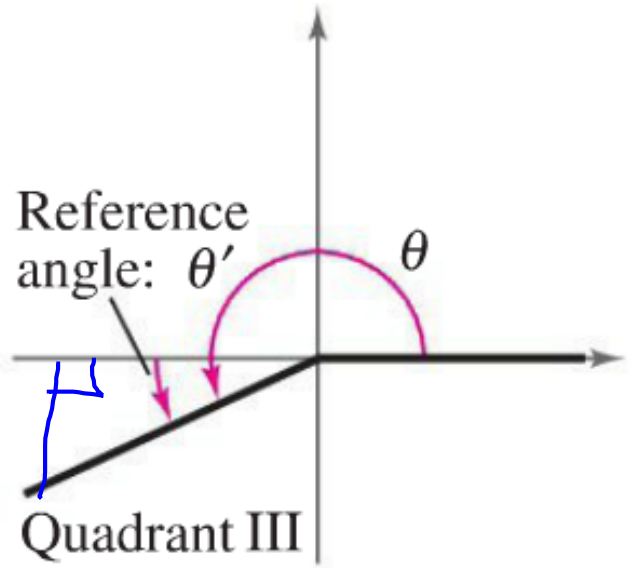
$\sqrt{5} = y$


S
A
y
Quadrant II
Quadrant I
Sin & Csc are positive
All are positive
x
Quadrant III
Quadrant IV
Tan & Cot are positive
Cos & Sec are positive
T
C

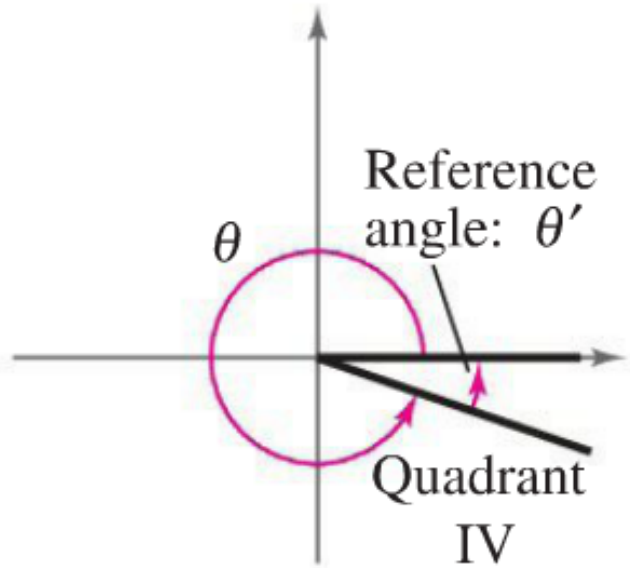
Reference Angles



$\theta' = \pi - \theta$ (radians)
$\theta' = 180° - \theta$ (degrees)



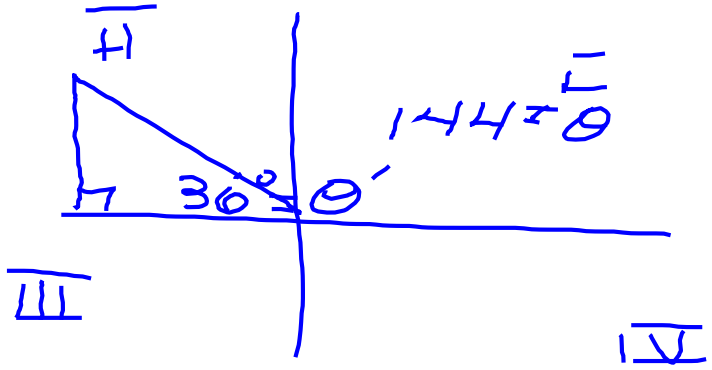
$\theta' = \theta - \pi$ (radians)
$\theta' = \theta - 180°$ (degrees)



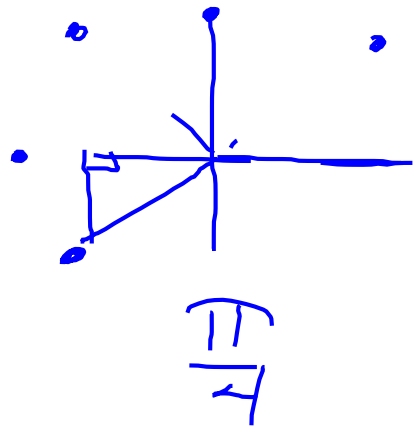
$\theta' = 2\pi - \theta$ (radians)
$\theta' = 360° - \theta$ (degrees)

Find the reference angle for each.

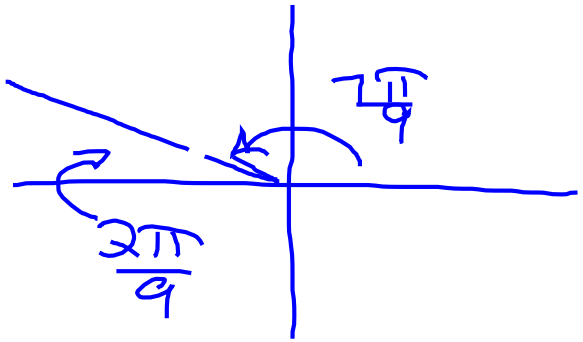
a. $\theta = 144°$



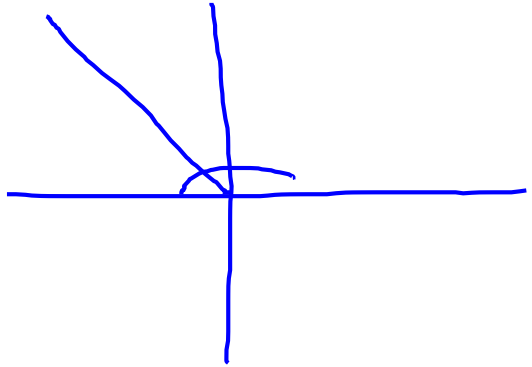
b. $\theta = \frac{5\pi}{4}$



c. $\theta = \frac{7\pi}{9}$

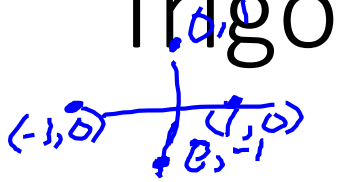


d. $\theta = 1.7$



3.14 − 1.7

Trigonometric Values of Common Angles



θ (degrees)	0°	30°	45°	60°	90°	180°	270°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und.	0	und

Evaluate with out using a calculator.

a. cos (-60°)

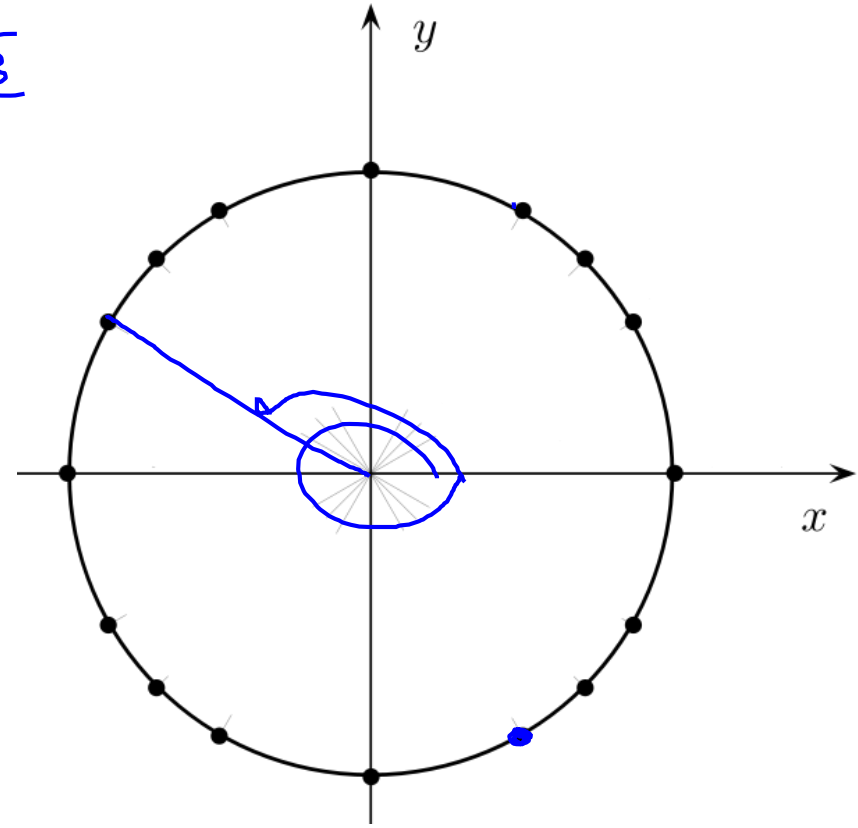
$\frac{1}{2}$

b. $\sin \frac{5\pi}{3}$

$-\frac{\sqrt{3}}{2}$

c. $\tan \frac{17\pi}{6}$

$-\frac{\sqrt{3}}{3}$



Let θ be an angle in Quadrant III such that $\sin\theta = -\frac{5}{13}$.

Find $\sec\theta$ and $\tan\theta$ using trig identities.

$y = -5$

$r = 13$

$\frac{r}{x} = \sec\theta = \frac{13}{12}$

$\tan\theta = \frac{-5}{12}$

$(13) = (\sqrt{x^2 + 25})$

$169 = x^2 + 25$

$144 = x^2$

$x = 12$

Use a calculator to evaluate each trig function.

a. cot 375 °

b. sin(-4.1)

c. sec $\frac{3\pi}{8}$

Section 4.4 p. 296; 1-29 odd, 45-67 odd