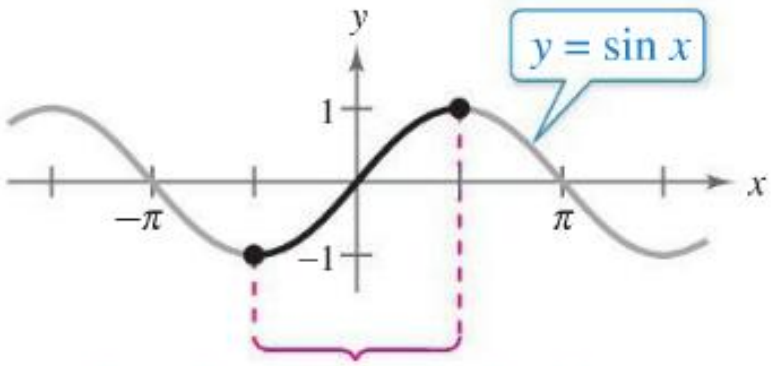
Inverse Trigonometric Functions

Lesson 4.7

Inverse Sine Function

To be one-to-one (pass the Horizontal Line Test) we must limit the domain of the inverse sine.



sin x has an inverse function on this interval.

The inverse sine function is denoted by $y = \arcsin x$ or $y = \sin^{-1} x$.

$(\sin x)^{-1}$ $\frac{1}{\sin x}$

Evaluating the Inverse Sine Function

If possible, find the exact value.

a. $\arcsin\left(\frac{1}{2}\right)$

30°

$\frac{\pi}{6}$

b. $\sin^{-1}\frac{\sqrt{3}}{2}$

60°

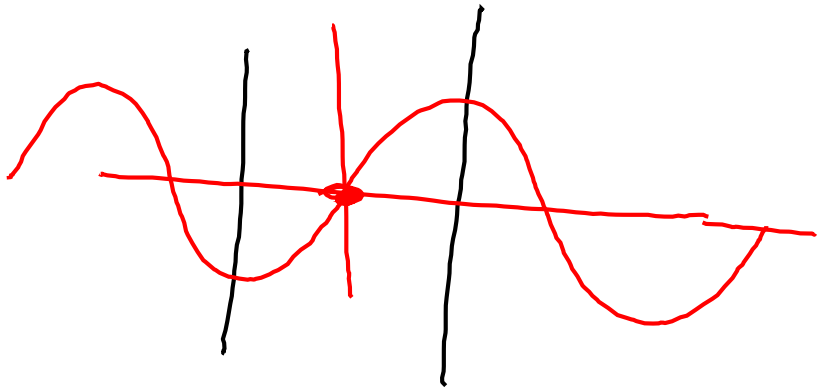
$\frac{\pi}{3}$

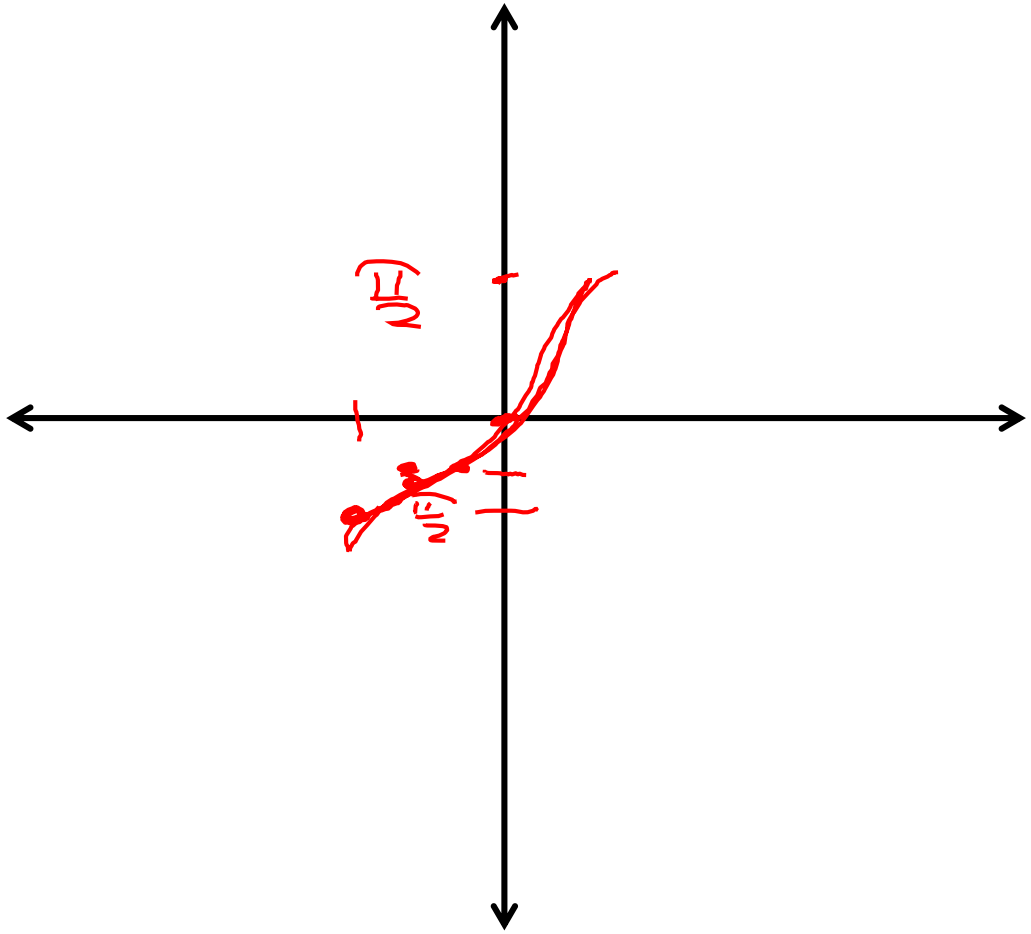
c. $\sin^{-1}\sqrt{3}$

not possible

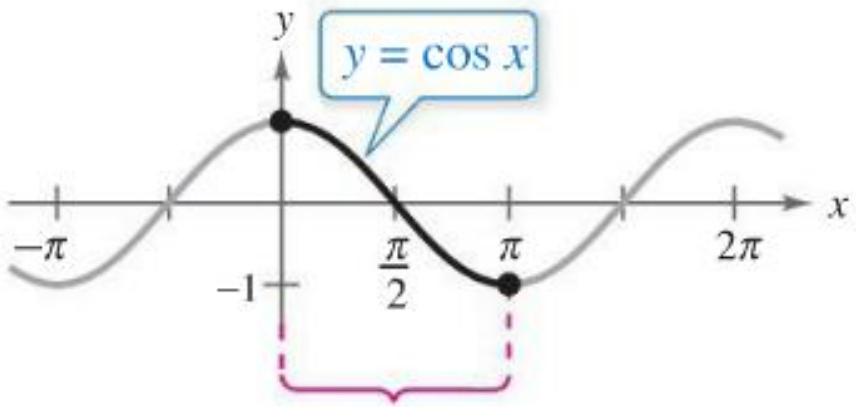
Graphing the Arcsine Function

y	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$x = \sin y$	-1	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	1





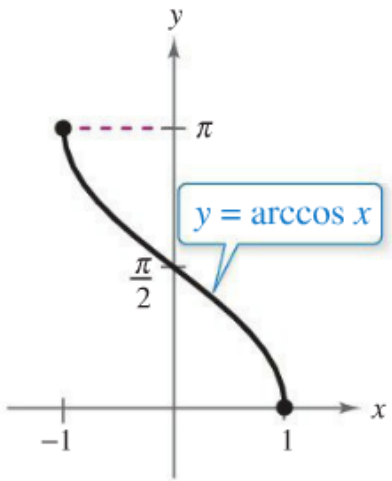
Other Inverse Trigonometric Functions



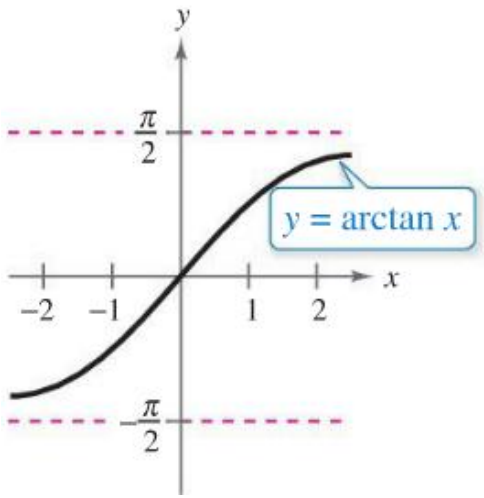
cos x has an inverse function on this interval.

Definitions of the Inverse Trigonometric Functions

Function	Domain	Range
$y = \arcsin x$ if and only if $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ if and only if $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ if and only if $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

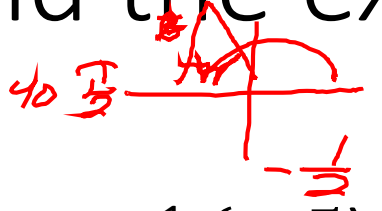


Domain: $[-1, 1]$
Range: $[0, \pi]$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

If possible, find the exact value.



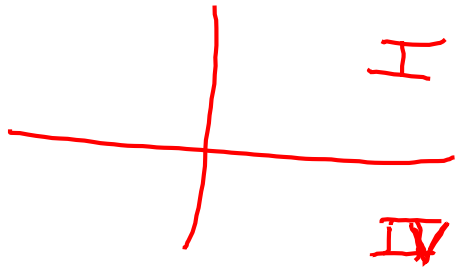
a. $\arccos\left(\frac{\sqrt{3}}{2}\right)$ 30° $\frac{\pi}{6}$

b. $\cos^{-1}(-.5)$ 120° $\frac{2\pi}{3}$

c. $\tan^{-1}\frac{\sqrt{3}}{3}$ 30° $\frac{\pi}{6}$

d. $\arctan(1)$ 45° $\frac{\pi}{4}$

	$\pi/6$ 30	$\pi/4$ 45	$\pi/3$ 60
sin	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$



Use a calculator to approximate the value, if possible to 3 decimal places.

$-\frac{\pi}{2}, \frac{\pi}{2}$

$0, \pi$

a. arctan 1.9

1.086

b. arccos $\left(-\frac{\pi}{3}\right)$

und

c. $\sin^{-1}(-0.919)$

-1.166

Composition of Functions

Inverse Properties of Trigonometric Functions

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$$\cos(\arccos x) = x \quad \text{and} \quad \arccos(\cos y) = y.$$

If x is a real number and $-\pi/2 < y < \pi/2$, then

$$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$

If possible, find the exact value.

a. tan(arctan 23)

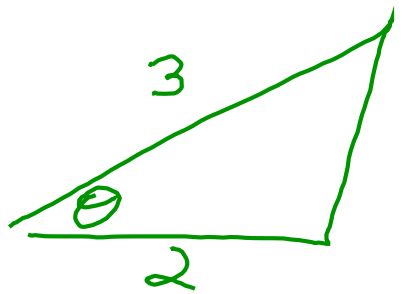
23

b. sin(arcsin π)

c. cos[arccos(-0.820]

−,820

Find the exact value of $\sin\left(\arccos\frac{2}{3}\right)$



.745

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