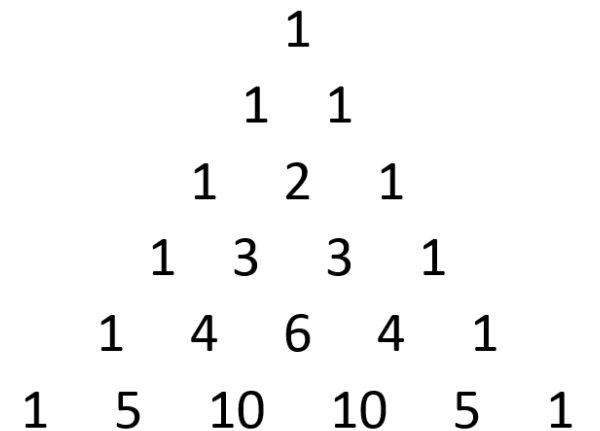


The Binomial Theorem

(and Pascal's Triangle) Lesson 9.5



Observations

$$(x + y)^0 = 1$$

$$(x + y)^1 = x + y$$

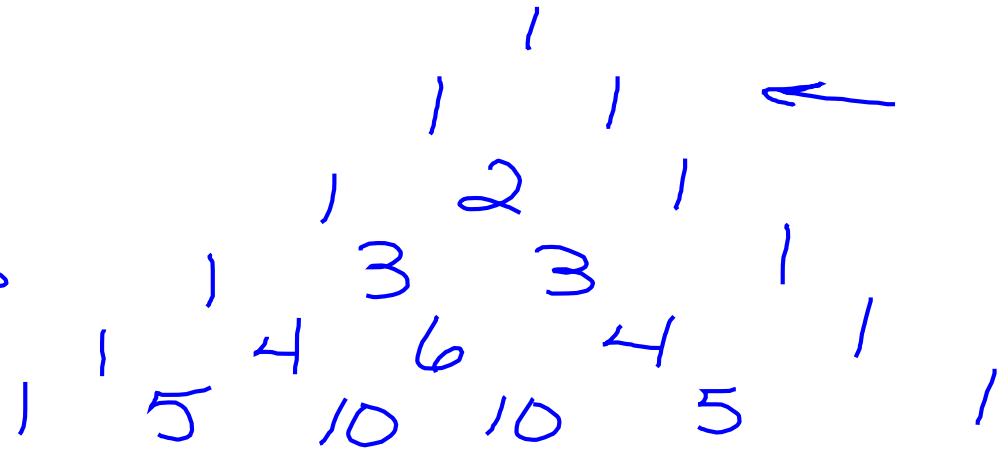
$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$(x+y)^n$ $n+1$ terms



Pascal's Δ

The Binomial Theorem

In the expansion of $(x + y)^n$

$$(x + y)^n = x^n + nx^{n-1}y + \cdots + {}_n C_r x^{n-r}y^r + \cdots + nxy^{n-1} + y^n$$

the coefficient of $x^{n-r}y^r$ is

$${}_n C_r = \frac{n!}{(n-r)!r!}.$$

The symbol $\binom{n}{r}$ is often used in place of ${}_n C_r$ to denote binomial coefficients.

Find the Binomial Coefficients

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

1. ${}_{10} C_5$

$$\frac{10!}{(10-5)!5!} = \frac{10!}{5!5!}$$
$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$$

2. ${}_{8} C_2 = 28$

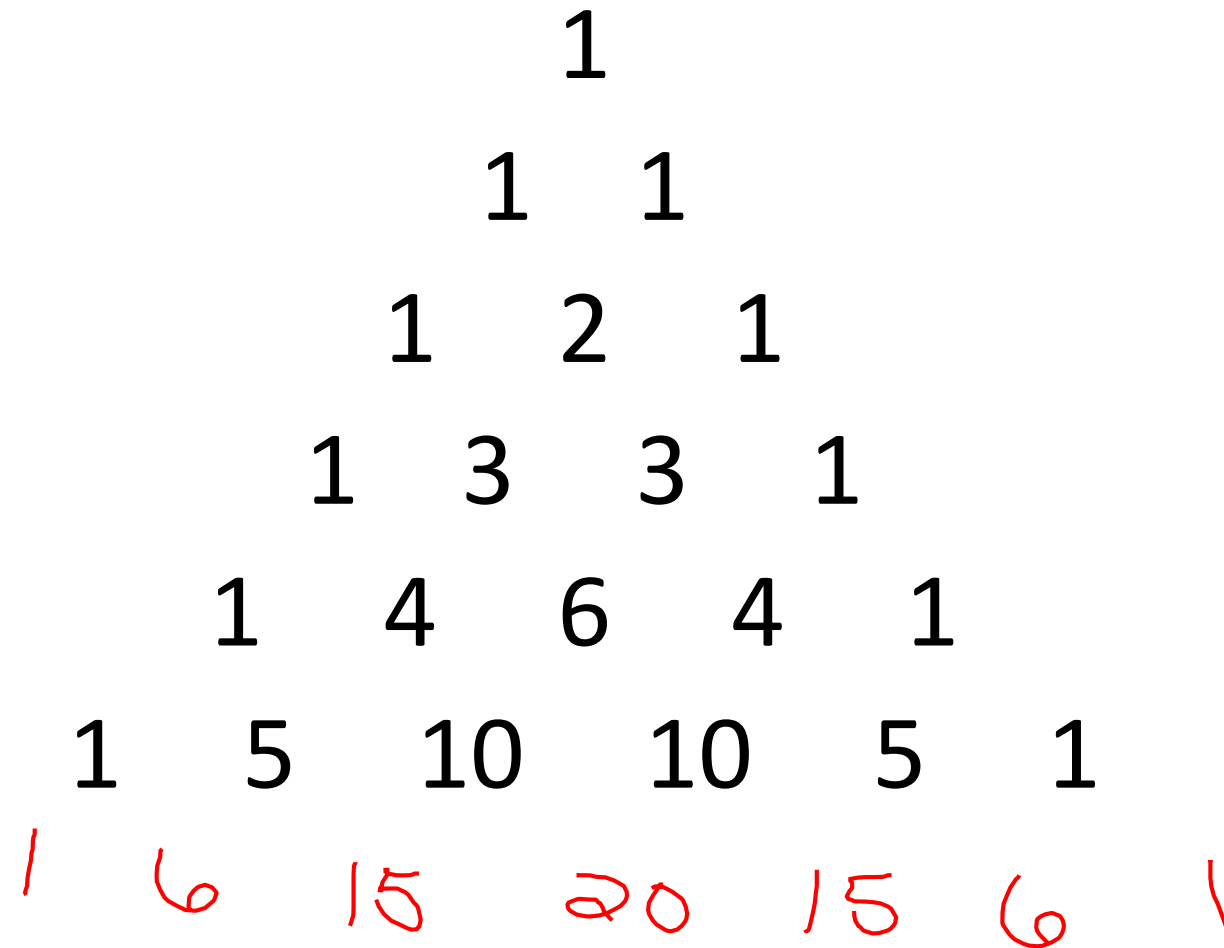
3. ${}_{12} C_4 = 495$

$$\frac{12!}{8!4!} = 495$$

4. ${}_{12} C_8 = 495$

$$\frac{12!}{4!8!} = 495$$

Pascal's Triangle



Find the expansion of $(x + 2)^4$.

$$(x + 2)^4 = \sum_{r=0}^4 {}_4C_r x^{4-r} 2^r$$

$${}_4C_0 x^4 (2)^0 + {}_4C_1 x^{4-1} 2^1 + {}_4C_2 x^2 (2)^2 + {}_4C_3 x^1 2^3 + {}_4C_4 x^0 2^4$$

$$\frac{4!}{4!0!} x^4 + \frac{4!}{3!1!} x^3 (2) + \frac{4!}{2!2!} x^2 (4) + \frac{4!}{1!3!} x^1 (8) + \frac{4!}{0!4!} (16)$$

$$1 x^4 + 8 x^3 + 24 x^2 + 32 x + 16$$

$$\begin{array}{cccccc} & & & & & 1 \\ & & & & & 1 & 1 \\ & & & & 1 & 2 & 1 \\ & & & 1 & 3 & 3 & 1 \\ & & 1 & 4 & 6 & 4 & 1 \\ 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

Find the expansion of $(3x - y)^4$.

$$(3x - y)^4 = \sum_{r=0}^4 {}^4C_r (3x)^{n-r} (-y)^r$$

$$\frac{4!}{1!3!} 3x(-y)^3$$

$$\begin{aligned}
 & {}^4C_0 (3x)^4 (-y)^0 + \underline{{}^4C_1} (3x)^3 (-y)^1 + {}^4C_2 (3x)^2 (-y)^2 + {}^4C_3 (3x)^1 (-y)^3 + \\
 & 81x^4 + \frac{4!}{3!1!} (27x^3)(-y) + \frac{4!}{2!2!} 9x^2 y^2 + {}^4C_4 (3x)^0 (-y)^4 \\
 & 81x^4 - 108x^3 y + 54x^2 y^2 - 12xy^3 + y^4
 \end{aligned}$$

$$\begin{array}{cccccc}
 & & & & & 1 \\
 & & & & & 4 \\
 & & & & & 1 \\
 & & & & & 1 & 1 \\
 & & & & & 1 & 2 & 1 \\
 & & & & & 1 & 3 & 3 & 1 \\
 & & & & & 1 & 4 & 6 & 4 & 1 \\
 & & & & & 1 & 5 & 10 & 10 & 5 & 1
 \end{array}$$

Find the coefficient of x^8 in the expansion of $(x^2 + 2)^{12}$.

$n=12$
 $r=8$

$$(x^2)^{12-r} = x^8$$

$$x^{24-2r} = x^8$$

$$24 - 2r = 8$$

$$-2r = -16$$

$$r = 8$$

$${}^{12}C_8 (x^2)^{12-8} (2)^8$$

$$\frac{12!}{4!8!} x^8 (256)$$

$$\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8}$$

$$11 \cdot 5 \cdot 9 (256) = 126,720$$

Find the fifth term of the expansion of $(2x - 3y)^{10}$.

Section 9.5 p. 649; 19-34 x 3's, 45-60 x 3's