The Inverse of a Square Matrix

Section 8.3

Identity Matrix

The n x n matrix with 1's on its main diagonal and 0's elsewhere is called the identity matrix of order n x n.

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Inverse of a Square Matrix

Let A be an n x n matrix and let I_n be the n x n identity matrix. If there exists a matrix A^{-1} such that

$$AA^{-1} = I_n = A^{-1}A$$

then A^{-1} is called the inverse of A. the symbol A^{-1} is read "A inverse".

Show that B is the inverse of A where

$$A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

1st column, 2nd column

1st row, 2nd row

$$\begin{bmatrix} 4\times4+5(-3) & 4(-5)+5(4) \\ (3)(4)+4(-3) & 3(-5)+4(4) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1st, 2nd, 3rd

1st row, 2nd, 3rd

Find the Inverse of a Matrix

Find the inverse of $A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$.

1. Let A be a square matrix of order n x n.
2. Write the matrix with the identity matrix on the right to get [A ⋮ I].
3. If possible, row reduce A to I using elementary row operations to get $[I \vdots A^{-1}]$.
4. Check your work by multiplying $AA^{-1} = I = A^{-1}A$.

Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 0 & -1 & 0 \end{bmatrix}$$

$-3R1+R2 \rightarrow$, $R3+R1 \rightarrow$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

$R3(2)+R2$: $0 \ -2 \ 0 \ 0 \ 0 \ 2$

$$\left[\begin{array}{cccccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{array}\right]$$

$0 \ 0 \ -1 \ +3 \ -1 \ -2$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 4 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 4 & -1 & -1 \\ 0 & 0 & -1 \\ -3 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} \ \end{bmatrix}$$

The Inverse of a 2 x 2 Matrix

A formula that works only for 2 x 2 matrices is given by

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if ad – bc $\neq$ 0. The inverse is given by

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. The denominator ad – bc is called the determinant of the matrix.

If possible, find the inverse of A = $\begin{bmatrix} 5 & -1 \\ 3 & 4 \end{bmatrix}$

Graphing Calculator to the Rescue!

You invest \$15,000 in AAA-rated bonds, AA-rated bonds and B-rated bonds, and want an annual return of \$1140. The average yields are 3.5% on the AAA bonds, 5% on the AA bonds and 6% on the B bonds. You invest twice as much on the AAA bonds as in the AA bonds.

$$\begin{cases} x + y + z = 15000 \\ .035x + .05y + .06z = 720 \\ x - 2y = 0 \end{cases}$$

How much have you invested in each? Use an inverse matrix to solve.

Lesson 8.3 p. 571; 5, 9, 14, 19, 20, 37, 45, 62