Matrices and Systems of Equations

Lesson 8.1

Definition of a Matrix

If m and n are positive integers, then an $m \times n$ (read "m by n") matrix is a rectangular array

$$\begin{array}{cc} & \begin{array}{ccccc} \text{Column 1} & \text{Column 2} & \text{Column 3} & \cdots & \text{Column } n \end{array} \\ \begin{array}{c} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \vdots \\ \text{Row } m \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

in which each **entry** a_{ij} of the matrix is a number. An $m \times n$ matrix has m rows and n columns. Matrices are usually denoted by capital letters.

Order of Matrices

A matrix having *m* rows and *n* columns is said to be of order m x n. Find the order of the following matrices.

a. $\begin{bmatrix} 7 & 3 \\ -1 & 5 \end{bmatrix}$ 2x2

b. $[-3]$ 1x1

c. $\begin{bmatrix} 1 & 0 \\ -3 & 6 \\ 5 & 1 \end{bmatrix}$ 3x2

d. $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ 2x3

Converting a System of Equations to an Augmented Matrix

System: $$\begin{cases} x - 4y + 3z = 5 \\ -x + 3y - z = -3 \\ 2x - 4z = 6 \end{cases}$$

Augmented Matrix: $$\left[\begin{array}{ccc:c} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{array}\right]$$

3x4

Coefficient Matrix: $$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$$

3x3

Write the augmented matrix for the system of linear equations.

$$\begin{cases} 5x + 9y - z = 8 \\ x - 4y + 3z = -6 \\ 7x + 12y + 7z = -3 \end{cases}$$

3×4

$$\left[\begin{array}{ccc|c} 5 & 9 & -1 & 8 \\ 1 & -4 & 3 & -6 \\ 7 & 12 & 7 & -3 \end{array}\right]$$

Elementary Row Operations

- Interchange two rows.
- Replace any row by a nonzero multiple of itself.
- Replace any row by the sum of itself and a multiple of any other row in the matrix.

Use an augmented matrix to solve.

$$\begin{cases} x + y + 2z = 3 \\ 3x + 4y + 4z = 9 \\ 5x + 2y + 15z = 13 \end{cases}$$

$-3(R1)+R2$

$-5(R1)+R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 3 & 4 & 4 & 9 \\ 5 & 2 & 15 & 13 \end{array}\right] \to \left[\begin{array}{cccc} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -3 & 5 & -2 \end{array}\right]$$

$R1 - R2 \to$

$+3(R2)+R3$

$$\left[\begin{array}{cccc} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -3 & 5 & -2 \end{array}\right] \quad \left[\begin{array}{cccc} 1 & 0 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & -2 \end{array}\right]$$

$2(R3)+R2$

$$\left[\begin{array}{cccc} 1 & 0 & 4 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$-4(R3)+R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z = 2$

$y - 4 = 0$

$y = 4$

$(-5, 4, 2)$

Solve the system

$$\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$$

$$\begin{array}{l} 3R2+R1 \\ R3+R2 \\ \\ \end{array}\left[\begin{array}{cccc} 3 & -2 & 1 & 15 \\ -1 & 1 & 2 & -10 \\ 1 & -1 & -4 & 14 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & -1 & -4 & 14 \\ 0 & 0 & -2 & 4 \\ 0 & 1 & 7 & -15 \end{array}\right]$$

$$\begin{array}{l} R1+R2 \\ -7R3+R2 \\ \\ \end{array}\left[\begin{array}{cccc} 1 & -1 & -4 & 14 \\ 0 & 1 & 7 & -15 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \left[\begin{array}{cccc} 1 & 0 & 3 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-3(R3)+R1$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & \sim \\ 0 & 1 & 0 & \sim \\ 0 & 0 & 1 & \sim \end{array}\right]$$

$(5, -1, -2)$

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