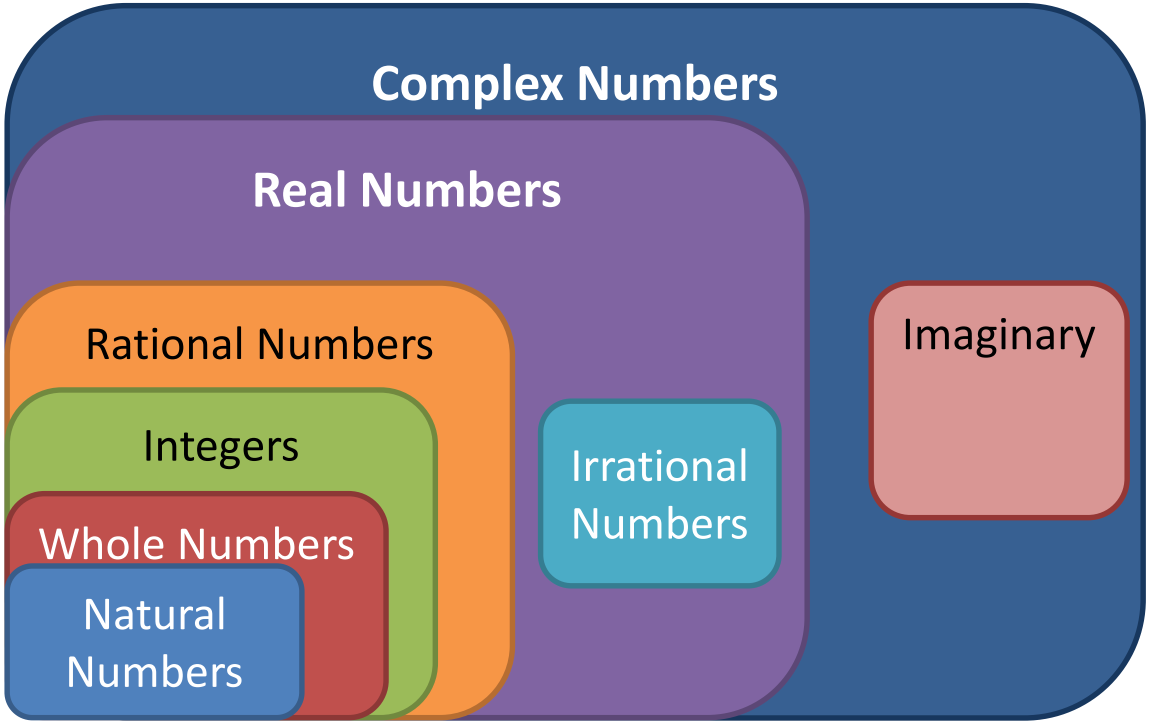
Zeros of Polynomial Functions

Section 2.5

Fundamental Theorem of Algebra

If f(x) is a polynomial of degree n, where n > 0, then f has at least one zero in the complex number system.

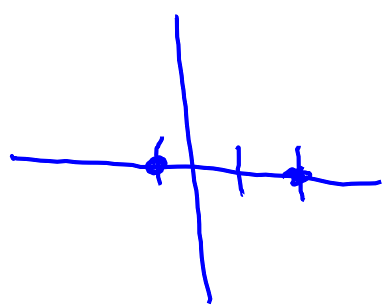


Linear Factorization Theorem

If f(x) is a polynomial of degree n, where n > 0, then f(x) has n zeros and n factors. (Not all will be unique. Remember multiplicity.)

If f(x) has zeros 2, -1, 4i, and -4i, then write the linear factors of f(x).

$(x-2)(x+1)(x-4i)(x+4i)$



Rational Zero Test

Solve $x^3 + 6x - 7 = 0$

q

p

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients, every rational zero of f has the form p/q, where p is a factor of a_0 and q is a factor of a_n. (factors of 7/1 in the above example.)

$\frac{-7}{1}$ $\qquad \frac{7, -7, 1, -1}{1, -1}$ $\qquad \frac{\pm 1 \; \pm 7}{\pm 1}$

Find the rational zeros of

$f(x) = x^3 - 7x - 6 = 0$

$\frac{-6}{1}$

$\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & -6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & -12 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & 1 & -1 & -6 & 0 \end{array}$$

$x^2 - x - 6$

$(x-3)(x+2)$

$x = -2, 3, -1$

Find the rational zeros of

$$f(x) = 3x^3 - 20x^2 + 23x + 10$$

Conjugate Pairs

Complex Zeros travel together.

Let f(x) be a polynomial function that has real coefficients. If **a + bi** where b ≠ 0 is a zero of the function, the conjugate **a – bi** will also be a zero of the function.

real + imag.

a + bi

2 ± 3i

Find a fourth degree polynomial with 0, 1, and i as zeros.

$-i$

$x\,(x-1)\,(x-i)(x+i)$

$x^2+xi-xi-i^2$

$(x^2-\cancel{x})(x^2+1)$

$x^4+x^2-x^3-x$

$f(x)=x^4-x^3+x^2-x$

Find all the zeros of

$f(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$

given $2 + 3i$ is a zero

$2-3i,\ \pm 1$

$$
\begin{array}{r}
x^2 - 1 \\
x^2-4x+13 \,\overline{\big)\, x^4 - 4x^3 + 12x^2 + 4x - 13} \\
-\,(x^4 - 4x^3 + 13x^2) \\
\hline
-x^2 + 4x - 13 \\
+x^2 - 4x + 13 \\
\hline
0
\end{array}
$$

$(x-2-3i)(x-2+3i)$

$$
\begin{array}{l}
x^2 - 2x + 3xi \\
\quad - 2x \qquad\qquad + 4 - 6i \\
\qquad\quad - 3xi \qquad + 6i + 9 \\
\hline
x^2 - 4x + 13
\end{array}
$$

$x^2 - 1 = (x-1)(x+1)$

- Section 2.5 p. 164 #s 19, 22, 30, 45, 47, 55, 59