Zeros of Polynomial Functions

Section 2.5 Day 2

Polynomial	Zeros	Changes in Sign	# of Positive Real Zeros
$6x^3 - 19x^2 + 16x - 4$	$2, \frac{1}{2}, \frac{2}{3},$	3	3
$X^3 - 5x^2 + 2x + 8$	$-1, 2, 4$	2	2
$2x^3 + 3x^2 - 8x + 3$	$1, \frac{1}{2}, -3$	2	2
$X^4 + 45x^2 - 196$	$2, -2, \pm 7i$	1	1
$X^5 + x^3 + 2x^2 - 12x + 8$	$1, 1, -2, \pm 2i$	2	2
$X^4 - 3x^3 + 6x^2 + 2x - 60$	$-2, 3, 1 \pm 3i$	3	1

Descartes' Rule of Signs

1. The number of positive real zeros of a function is either equal to the number of variations in signs of the function or less than that number by an even integer.
2. The number of negative real zeros is either equal to the number of variations in sign of f(-x) or less than that number by an even integer.

Describe the possible zeros of

$f(x) = 6x^3 - 19x^2 + 16x - 4$

3 pos or 1 pos

$f(-x) = 6(-x)^3 - 19(-x)^2 + 16(-x) - 4$

$-6x^3 - 19x^2 - 16x - 4$

0 neg zeros

Upper and Lower Bound Rules

Let f(x) be a polynomial with real coefficients and a positive leading coefficient. Suppose f(x) is divided by x – c using synthetic division.

- If $c > 0$ and each number in the last row is either positive or zero, then c is an upper bound for the real zeros of f. (Don't try anything higher.)
- If $c < 0$ and the numbers in the last row are alternately positive and negative (zero entries count as either positive or negative), then c is a lower bound for the real zeros of f. (Don't try anything lower.)

$f(x) = 4x^5 + 6x^2 - 7x + 3$

$F(1) = 4$

0

Put it All Together

$f(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$

4 zeros

$\frac{\text{factors of } 4}{\text{factors of } 1}$ $\pm 1, \pm 2, \pm 4$

pos reals 4 or 2 or 0

$f(-x) = (-x)^4 - 2(-x)^3 + 5(-x)^2 - 8(-x)$

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1, 2, 4 possible zeros

	1	-2	5	-8	4
1		1	-1	4	-4
	1	-1	4	-4	0
2	1	0	5	2	8

$1, 1 \pm 2i$

Section 2.5 p. 165 #s 90, 91, 93, 67, 74, 76, 77, 82