Polynomial and Synthetic Division

Section 2.3

Do Now – by Hand (no Calculator)

23
12) 276
-24
36
-36
0

88 R1
4
45) 3960
8 -360
360
-360

76) 7524

Long Division of Polynomials

Divide $2x^3 - 5x^2 + x - 8$ by $x - 3$

$$2x^2 + x + 4 + \frac{4}{x-3}$$

$$x - 3 \overline{\smash{)}\, 2x^3 - 5x^2 + x - 8}$$

$$-2x^3 + 6x^2$$

$$x^2 + x$$

$$-x^2 + 3x$$

$$4x - 8$$

$$-4x + 12$$

$$+4$$

Divide $2x^4 + 4x^3 - 5x^2 + 3x - 2$ by $x^2 + 2x - 3$

$$2x^2$$

$$x^2 + 2x - 3 \overline{\big)\, 2x^4 + 4x^3 - 5x^2 + 3x - 2}$$

$$\underline{2x^4 + 4x^3 - 6x^2}$$

Divide $x^3 - 1$ by $x - 1$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x - 1 & x^3 + 0x^2 + 0x - 1 \\
 & -x^3 + x^2 \\
\hline
 & x^2 + 0x \\
 & -x^2 + x \\
\hline
 & x - 1 \\
 & -x + 1 \\
\hline
 & 0
\end{array}
$$

Division Algorithm

For all polynomials f(x) and d(x) such that the degree of d is less than or equal to the degree of f and d(x) $\neq$ 0, there exists unique polynomials a(x) and r(x) such that f(x) = d(x)q(x) + r(x) where r(x) = 0 or the degree of r is less than the degree of d.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Synthetic Division

Only works with a divisor of form $x - k$.

Divide $3x^3 - x^2 + 2x - 3$ by $\underline{x - 2}$

$x - 2 = 0$

$x = 2$

$$
\begin{array}{r|rrrr}
2 & 3 & -1 & 2 & -3 \\
 & \downarrow & 6 & 10 & 24 \\
\hline
 & 3 & 5 & 12 & \multicolumn{1}{|r}{21}
\end{array}
$$

$$3x^2 + 5x + 12 + \frac{21}{x-2}$$

Remainder Theorem

If a polynomial f(x) is divided by x – k, the remainder is f = f(k).

$2x^3 - 8x^2 - 3x - 9 \div x + 1$

$$
\begin{array}{r|rrrr}
-1 & 2 & -8 & -3 & -9 \\
 & \downarrow & -2 & 10 & -7 \\
\hline
 & 2 & -10 & 7 & \big| \; -16
\end{array}
$$

f(-1) = -16

Factor Theorem

A polynomial f(x) has a factor x – k, if and only if f(k) = 0.

Show $x - \frac{1}{2}$ is a factor of $f(x) = 2x^3 - 15x^2 + 27x - 10$

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & \downarrow & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$2x^2 - 14x + 20$

Find the other factors of $2x^3 - 15x^2 + 27x - 10$

$2(x^2 - 7x + 10)$

$(x - 1/2)(x - 5)(x - 2)$

$1/2, 5, 2$

Uses of the Remainder Theorem

1. The remainder r gives the value of f at $x = k$.
2. If $r = 0$, $(x - k)$ is a factor of $f(x)$
3. If $r = 0$, $(k, 0)$ is an x-intercept of the graph of f.

$x^4 - 4x^3 - 15x^2 + 58x - 40$ Verify $(x - 5)$ and $(x + 4)$ are factors.

$$
\begin{array}{r|rrrr|r}
5 & 1 & -4 & -15 & 58 & -40 \\
 & \downarrow & 5 & 5 & -50 & 40 \\
\hline
-4 & 1 & 1 & -10 & 8 & ☺ \\
 & \downarrow & -4 & 12 & -8 & \\
\hline
 & 1 & -3 & 2 & ☺ &
\end{array}
$$

$x^2 - 3x + 2$

$(x - 2)(x - 1)$

Factors – $(x-5)(x+4)(x-2)(x-1)$

Zeros - $x = 5, -4, 2, 1$

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11, 13, 24, 27, 36, 47, 51, 59, 62, 67, 70

Study for Quiz on 2.1 – 2.3