Inverse Functions

Section 1.9

Review

- Notation: f^{-1} or $f^{-1}(x)$ – inverse not reciprocal
- A pair of functions such that:
    - the domain of $f(x)$ is the range of $f^{-1}(x)$
    - the range of $f(x)$ is the domain of $f^{-1}(x)$
- The functions "undo" each other. So $f(f^{-1}(x) = x$ and $f^{-1}(f(x)) = x$. Composing them both ways results in x.

Is g(x) inverse of f(x) = 5x - 8?

- g(x) = $\dfrac{x-8}{5}$

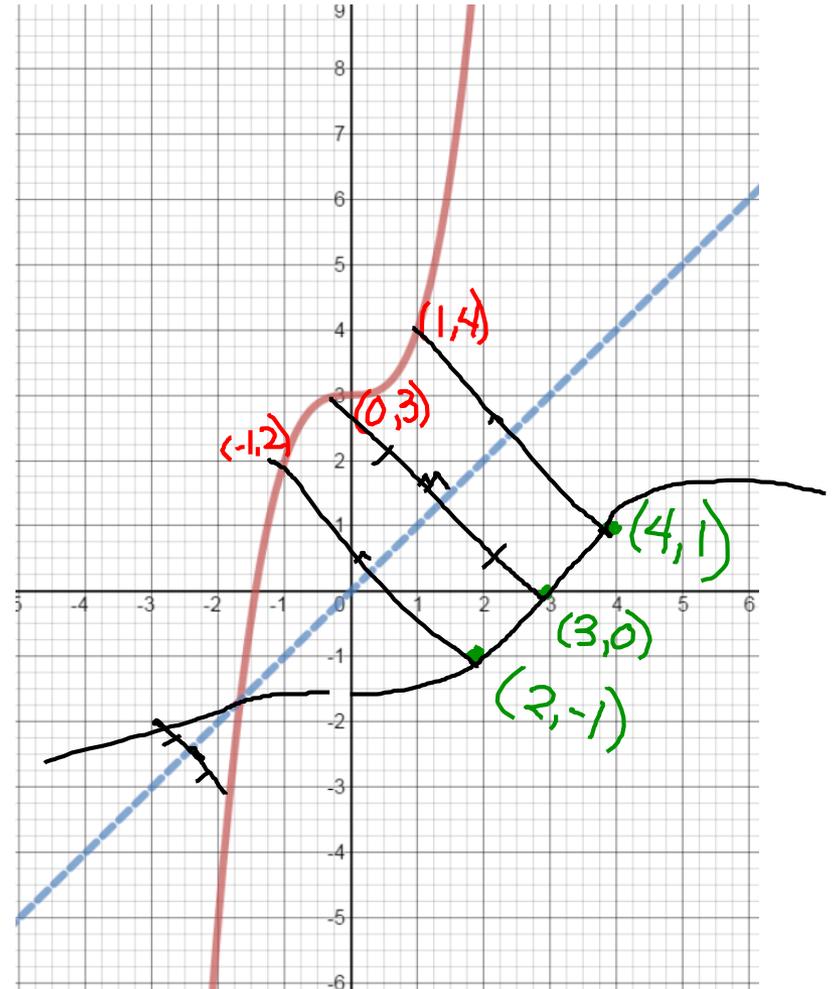$(f \circ g)(x) = 5\left(\dfrac{x-8}{5}\right) - 8$

$= x - 8 - 8$

$= x - 16$

not inverses

Graphs of Inverse Functions

- If (x, y) lies on the graph of f, then (y, x) must lie on the graph of f^{-1}.

- The graph of f^{-1} is a reflection of the graph of f in the line y = x.

Finding Inverse Functions

- Replace f(x) with y.

- Switch x and y.

- Solve for y.

- Replace y with $f^{-1}(x)$.

- Find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$ to verify they both equal x.

Find the inverse of f(x) = -2x + 3.

$$f(x) = -2\left(-\frac{1}{2}x + \frac{3}{2}\right) + 3$$

$$+x - 3 + 3$$

$$= x$$

$$y = -2x + 3$$

$$x = -2y + 3$$

$$\frac{x-3}{-2} = \frac{-2y}{-2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$f^{-1}(x) = -\frac{1}{2}x + \frac{3}{2}$$

New – More Interesting functions

Find the inverse of $f(x) = x^3 + 4$

$$y = x^3 + 4$$

$$x = y^3 + 4$$

$$\sqrt[3]{x - 4} = \sqrt[3]{y^3}$$
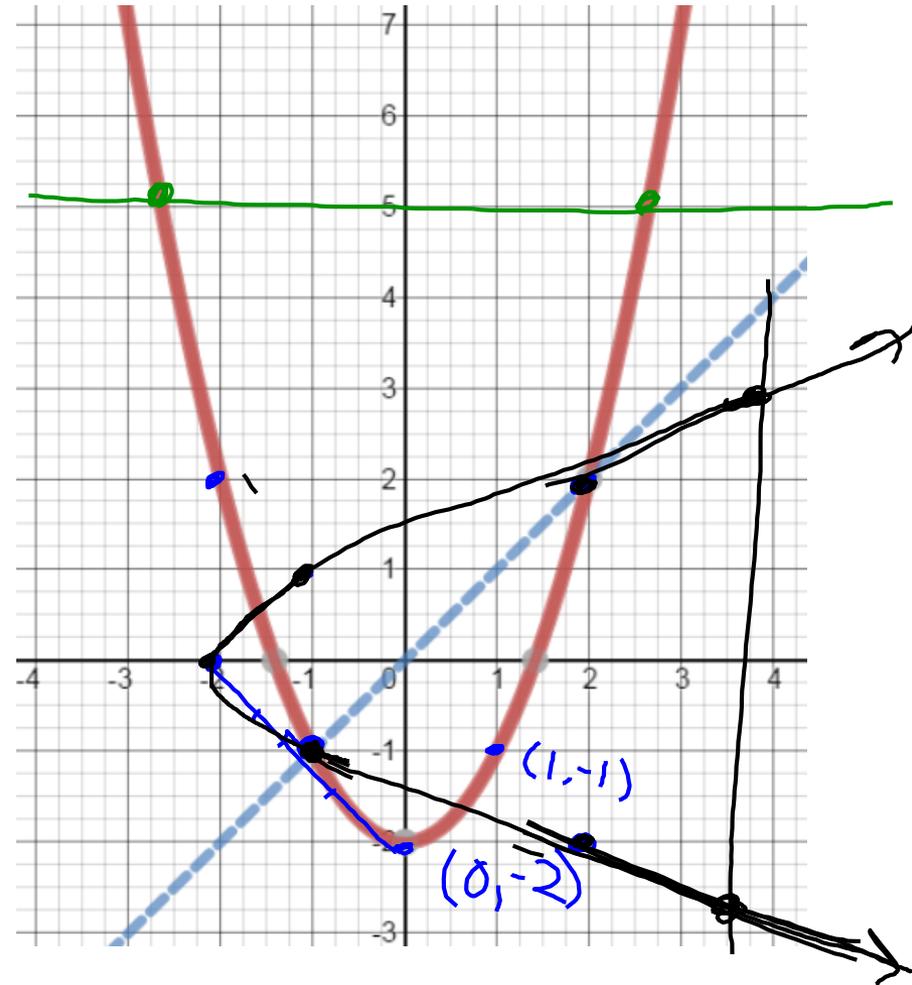
$$y = \sqrt[3]{x - 4}$$

$$f^{-1}(x) = \sqrt[3]{x - 4}$$

D $(-\infty, \infty)$

R $(-\infty, \infty)$

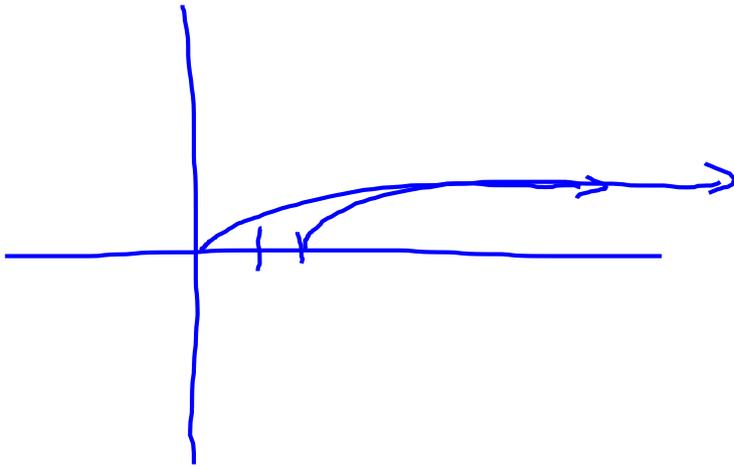D $(-\infty, \infty)$

R $(-\infty, \infty)$

Is the inverse a function?

Horizontal Line Test

- Tells you if your function has an inverse.

- One-to-One Functions – each value of the dependent variable corresponds to exactly one value of the independent variable. A function has an inverse only if the function is one-to-one.

Find the inverse of $f(x) = \sqrt{x-2}$

$$y = \sqrt{x-2}$$

$$(x)^2 = \left(\sqrt{y-2}\right)^2$$

$$x^2 = y-2$$

$$f^{-1}(x) = x^2+2$$

$D.$ $[2, \infty)$ $\mathbb{R}$ s.t $x \geq 2$

$R.$ $[0, \infty)$ $\mathbb{R}$ s.t $y \geq 0$

Identify the domain and range of the inverse function.

Find the inverse of $f(x) = \dfrac{x-1}{x+3}$

$$y = \frac{x-1}{x+3}$$

$$x = \frac{y-1}{y+3}$$

$$x(y+3) = y-1$$

$$xy + 3x = y-1$$

$$\frac{-y \quad -3x \quad -y \quad -3x}{}$$

$$xy - y = -3x-1$$

$$y(x-1) = -3x-1$$

$D: (-\infty, -3) \cup (-3, \infty)$

R St $x \neq -3$

$$y = \frac{-3x-1}{x-1}$$

$$f^{-1}(x) = \frac{-3x-1}{x-1}$$

D R St $x \neq -3$ or $x \neq 1$

p. 90: 13, 15, 17-20, 37-40
48, 51, 53 – no part b on these three,
64, 70